The Lucas-critique
Reference note to lecture 8 in ECON 5101, Time Series Econometrics

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1 Introduction

Lucas (1976), together with Sims (1980), are two of the most influential and "formative" papers in the field of modern applied macroeconomics and econometrics. We referred to Sim's paper in the VAR lecture (#7). Lucas' offering from 1976 contains what has become known as the *Lucas-critique*. This note contains a brief exposition, and notes the line back to Haavelmo (1944), and to the econometric concept of super exogeneity.

2 Invariance and the Lucas-critique

The essence in the Lucas-critique can be shown by starting with the simple relationship

$$y_t = \beta_0 x_t + \eta_t, \tag{2.1}$$

which we assume is estimated by OLS. η_t denotes a disturbance whose properties depend on the data generating process (DGP), which is assumed to be

$$y_t = \beta_0^* E(x_t \mid \mathcal{I}_{t-1}) + \varepsilon_t \tag{2.2}$$

$$x_t = a_{22} x_{t-1} + \epsilon_{xt}, \tag{2.3}$$

where $-1 < a_{22} < 1$. ε_t and ϵ_{xt} are independent white-noise processes. $\mathcal{I}_{t-1}$ denotes the *information set* used to form rational expectations. In this simple case we can replace $E(x_t \mid \mathcal{I}_{t-1})$ by the more practical notation $E(x_t \mid x_{t-1})$. By assumption we also have

$$E(\varepsilon_t \mid x_t) = 0.$$

Without loss of generality, assume that ε_t and ϵ_{xt} are independent and normally distributed. y_t is then a normal variable with mean:

$$E(y_t \mid x_{t-1}) = \alpha_{22}\beta_0^* x_{t-1},$$

and x_t has also a normal distribution with mean

$$E(x_t \mid x_{t-1}) = a_{22} x_{t-1}.$$

The conditional mean of y_t, given x_t, becomes

$$E(y_t \mid x_t) \equiv \mu_{Y|X} = \beta_0^* x_t + E(\eta_t \mid x_t), \tag{2.4}$$

where $E(\eta_t \mid x_t) \neq 0$, since x_t must be correlated with η_t due to

$$\eta_t = \varepsilon_t - \beta_0^* \epsilon_{xt}$$

and the DGP (2.2) and (2.3). Specifically, we have

$$E(\eta_t \mid x_t) = -\beta_0^* E(\epsilon_{xt} \mid x_t) \tag{2.5}$$

since $E(\varepsilon_t \mid x_t) = 0$ from the assumptions of the DGP.

With reference to the assumed normality we have the regression function:

$$E(\epsilon_{xt} \mid x_t) = \delta x_t$$

where

$$\delta = \frac{E(\epsilon_{xt} x_t)}{Var(x_t)} = \frac{\sigma^2_{\epsilon_{xt}}}{Var(x_t)}$$

and where $\sigma^2_{\epsilon_{xt}}$ denotes the variance of ϵ_{xt}. Subject to stationarity we have:

$$Var(x_t) = \frac{\sigma^2_{\epsilon_{xt}}}{1 - a_{22}^2}$$

The regression coefficient δ is therefore

$$\delta = (1 - a_{22}^2). \tag{2.6}$$

Substitution into (2.4) of the expressions for $E(\eta_t \mid X_t)$, (2.5) and δ, (2.6), give the conditional expectation for y_t when the DGP is characterized by rational expectations, (2.2) and (2.3):

$$\mu_{Y|X} = \beta_0^* x_t - \beta_0^*(1 - a_{22}^2) x_t = (\beta_0^* a_{22}^2) x_t. \tag{2.7}$$

The OLS estimator $\hat{\beta}_0$ is as always consistent for the regression coefficient between y_t and x_t, but in this case that coefficient is not a single parameter but $\beta_0^* a_{22}^2$. Hence we get

$$\text{plim}(\hat{\beta}_0) = \beta_0^* a_{22}^2 < \beta_0^*, \text{ since } -1 < a_{22} < 1. \tag{2.8}$$

However, the real thrust of the Lucas-critique is that *any* change in the formation of expectations, here represented by a_{22}, is predicted to lead to a change in $\text{plim}(\hat{\beta}_0)$. Therefore the estimator of $\hat{\beta}_0$ gives the wrong conclusions about how a policy change affects y_t.

We say that the parameter β_0 is not invariant to changes in expectations (changes in a_{22}) and therefore x_t in (2.1) is not super exogenous when the DGP contains rational expectations as in (2.2) and (2.3).

In contrast to β_0, the parameter β_0^* is often called a "deep-structure parameter". Given the premis of the the β_0^* *is* invariant to interventions in the x_t process when the true DGP is (2.2) and (2.3).

3 The Lucas-critique as a possibility theorem—not a truism

The Lucas critique identifies one mechanism that can induce lack of invariance (and a low degree of autonomy) in the parameters of a conditional econometric model. In practice there are many others pitfalls as well. Omitted variables, for example, is a important (though more "mundane") cause of lack of invariance in econometric models, including those estimated by IV/GMM.

The exact set of assumptions is important when discussing invariance. Assume for example that another theory than the rational expecations model is valid, namely

$$y_t^p = \beta_0 x_t$$

where y_t^p is the agents' planned consumption, or price change. If $y_t = y_t^p + \varepsilon_t$ and $E(\varepsilon_t, x_t) = 0$, then $\text{plim}(\hat{\beta}_0) = \beta_0$ which is the relevant parameter for policy evaluation. Hence, the force of the Lucas critique depends on the relevance of the underlying theory, which needs to be tested on its own ground.

4 Empirical testing of the Lucas critique

One approach to testing the force of the Lucas-critique is to check whether the predicted invariance occurs in the conditional model after a significant structural break in the marginal part of the model. We refer to this as tests of invariance (and of super exogeneity), see e.g., Engle et al. (1983), Engle and Hendry (1993).

In terms of the *conditional-plus-marginal* model of the VAR in Lecture 7: If there is a structural break in the marginal equation for x_t, it is quite likely that one or more of the parameters of the conditional ADL model for y_t will change in the period of the break (or before/after?). However, such a change (lack of invariance) does not follow logically. Invariance of parameters, and thus autonomy with respect to the breaks tested for, is also a possible outcome of the structural-break "event".

In practice, recursive estimation of the parameters of the conditional and marginal models can be used as an informal test.

Formal tests are also quite easy to perform. For example, we can represent structural breaks in the marginal model by impulse or step dummies. If these dummies are jointly significant F-statistic when added to the conditional model, the null hypothesis of invariance (super-exogeneity) can be rejected (The Lucas critique is confirmed).

As usual, failure to reject the null hypothesis does not imply that it is true. Specifically, it does not mean that expectations are not important. It can be that Lucas critique lacks force. It may be that the rational expectations model is too specific about how expectations are formed and carried trough to economic decisions.

5 The Lucas-critique and the Haavelmo-critique

Early in the history of econometrics Frisch and Haavelmo spent a lot of time thinking about model properties like autonomy, confluence and invariance. The discussion in section 8 of Haavelmo (1944) is an interesting read, in particular with the Lucas-critique in mind.

About the same time as Lucas' break-through, Leif Johansen also took on board the concept of (degree of) autonomy in his theory of economic planning, see Johansen (1977, Ch 4.6).

References

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