

Exam 5106/9106 – Fall 2019

1. Assume unemployment duration t_i has the following distribution:

$$f(t; \theta) = (1/\theta) \exp(-t/\theta)$$

with $E[t] = \theta$.

- (a) Write down the log-likelihood for an i.i.d. sample of size n .

$$\mathcal{L} = -\sum_{i=1}^n (t_i/\theta + \log(\theta)) = -n\bar{t}/\theta - n\log(\theta)$$

- (b) Derive the score.

$$\frac{\partial \mathcal{L}}{\partial \theta} = -\sum_{i=1}^n \left(-\frac{t_i}{\theta^2} + \frac{1}{\theta}\right) = n\bar{t}/\theta^2 - n/\theta$$

- (c) Derive the $\hat{\theta}_{MLE}$, the ML estimator of θ .

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0 \Rightarrow n\bar{t}/\theta^2 - n/\theta = 0 \Rightarrow \hat{\theta}_{MLE} = \bar{t}$$

- (d) What is the variance of $\hat{\theta}_{MLE}$?

$$Var(\hat{\theta}_{MLE}) = \theta^2/n \text{ since } Var(\hat{\theta}_{MLE}) = -\frac{1}{n}E[\partial^2 \mathcal{L}/\partial \theta^2]^{-1} \text{ and}$$

$$E[\partial^2 \mathcal{L}/\partial \theta^2] = E[2t_i/\theta^3 - 1/\theta^2] = -2E[t]/\theta^3 + 1/\theta^2 = -1/\theta^2$$

Suppose $n = 4$ and t takes on the following values in your sample: 4.2, 5.3, 8.1, 14.4.

- (e) What is the MLE estimate of θ ? What is its standard error?

$$\hat{\theta}_{MLE} = (4.2 + 5.3 + 8.0 + 14.2)/4 = 8 \text{ and s.e.} = 8/2 = 4$$

- (f) Test for $H_0 : \theta = 3$ against $H_1 : \theta \neq 3$ at the 5% significance level using the LR, Wald and LM test.

$$LR = 2 * (-4 * (1 + \log(8)) - -4 * (8/3 + \log(3))) = 5.49,$$

$$W = (8 - 3)^2 / (8^2/4) = 25/16 = 1.5625,$$

$$LM = (4 * 8/3^2 - 4/3)^2 * (3^2/4) = 11.11. \text{ LR and LM tests are significant at the 5\% level.}$$

2. Apply a regression discontinuity design with running variable x and threshold 0.5 on the following two datasets

Dataset A			Dataset B		
x	d	y	x	d	y
0.469	0	0.758	0.469	0	0.758
0.471	0	0.532	0.471	0	0.532
0.479	0	0.907	0.479	1	0.907
0.490	0	0.542	0.490	0	0.542
0.497	0	0.699	0.497	0	0.699

0.505	1	2.312		0.505	1	2.312	
0.515	1	2.113		0.515	0	2.113	
0.521	1	1.884		0.521	1	1.884	
0.525	1	2.015		0.525	1	2.015	
0.527	1	1.903		0.527	1	1.903	
+-----+				+-----+			

Use Dataset A

- (a) Does this look like a sharp or a fuzzy design? Explain.

Sharp design. Everybody is treated $\geq .5$ and nobody is treated $< .5$

- (b) Estimate and interpret the effect of d on y using local constant regression, a uniform (box) kernel and a bandwidth of 0.02.

$(2.312 + 2.113)/2 - (.542 + .699)/2 = 2.2125 - .6205 = 1.592$. Treatment increases Y by ca. 1.6

- (c) Estimate and interpret the effect of d on y using local linear regression, a uniform (box) kernel and a bandwidth of 0.02.

$E[y^0|x=.5] = 0.542 + (.5 - .49) * (.699 - .542) / (.497 - .490) = .76628571$
 $E[y^1|x=.5] = 2.113 + (.5 - .515) * (2.113 - 2.312) / (.515 - .505) = 2.4115$
Treatment increases Y by ca. $2.4115 - .7662857 = 1.6452143$

- (d) Estimate the bias of local constant regression for $E[y^0]$ at $x = 0.5$.

Bias = estimate - unbiased estimate = $.6205 - .76628571 = -.14578571$

Use dataset B

- (e) Estimate and interpret the effect of d on y using local constant regression, a uniform (box) kernel and a bandwidth of 0.06.

$(E[y|x \geq .5] - E[y|x < .5]) / (E[d|x \geq .5] - E[d|x < .5]) = (2.0454 - .6876) / (4/5 - 1/5) = 2.263$.

This is the average effect of d on y for people with $x=.5$ whose treatment status changes when crossing the threshold, i.e. a LATE at $x=.05$

- (f) Assume x is exogenous. Compute $E[y^0|\text{never taker}]$ and $E[y^1|\text{complier}]$.

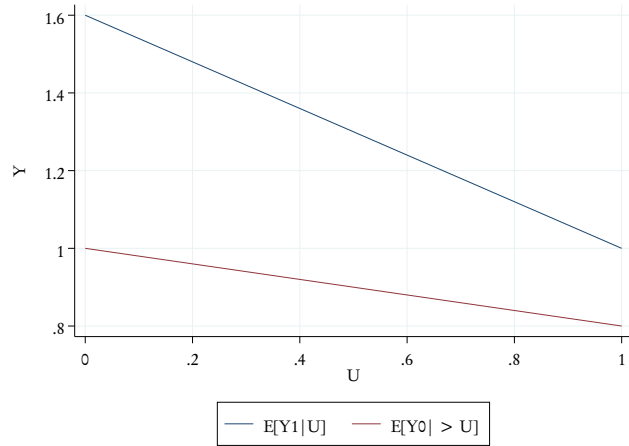
$E[y^0|\text{never taker}] = E[y|d=0, z=1] = 2.113$
 $E[y^1|\text{complier}] = \{E[y|d=1, z=1] - E[y|d=1, z=0] * pa / (pa + pc)\} * (pa + pc) / pc$
 $pc = (4/5 - 1/5) = 3/5$, $pa = P[d=1|z=0] = 1/5$.
 $E[y^1|\text{complier}] = (2.0285 - 0.907 * 1/4) * (4/3) = 2.4023333$

3. You are interested in how a binary treatment D affects an outcome Y . You have a continuous instrument Z_c that has full support. Treatment is assigned as follows

$$D = \begin{cases} 1 & Z_c - V > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $Z_c, V \sim N(0, 1)$. Let $U \equiv \Phi(V)$ where $\Phi(\cdot)$ denotes the CDF of the standard normal distribution.

Also assume the following potential outcome conditional expectation functions:



(a) Draw $\Pr(D = 1|U)$.

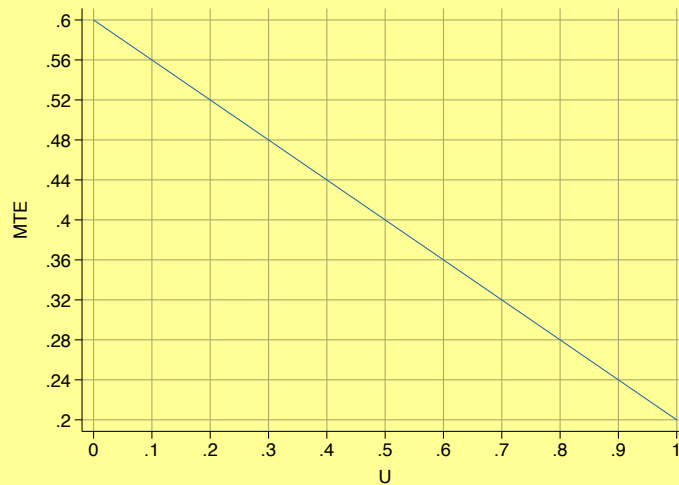


since

$$\begin{aligned}
 P(D = 1|U = u) &= P(D = 1|\Phi(V) = u) \\
 &= P(Z - \Phi^{-1}(u) > 0) \\
 &= P(Z > \Phi^{-1}(u)) \\
 &= 1 - \Phi(\Phi^{-1}(u)) \\
 &= 1 - u
 \end{aligned}$$

- (b) Draw the MTE function and explain its interpretation.

this is the difference between $E[y_1|u]$ and $E[y_0|u]$



- (c) Considering equation (1) and your answer in 3(b), does selection into the treatment make economic sense? Explain.

Those with the highest (latent) propensity to select into treatment (low U) have the highest returns (MTE). This is consistent with an economic choice model where people select treatment based on gains in terms of Y . If people care about other outcomes then gains in Y could be offset by losses elsewhere in which case the observed selection would not be economically consistent with the net benefits of the treatment.

- (d) What is the ATE given the MTE function in (b)?

Given that U is uniform, the average MTE equals $(0.6 + 0.2) / 2 = 0.4$.

Suppose you have a binary instrument $Z = \begin{cases} 1 & Z_c = z \\ 0 & Z_c = z' \end{cases}$ for which 20% of the people are treated if $Z = 0$ while 30% is treated if $Z = 1$. Assume $P(Z = 1) = 0.4$.

- (e) What is the LATE given the MTE function in (b)?

The LATE is the average MTE over $U \in [0.2, 0.3]$, i.e. the LATE = 0.5.

- (f) What is the ATT given the MTE function in (b)? [Hint: The treated are a combination of compliers and always-takers]

Note that $P(D = 1) = P(\text{always-taker}) + P(\text{complier})P(Z = 1)$. The ATT is thus a weighted average of the effect for always takers ($U \in [0.8, 1]$) which is 0.24, and the LATE, 0.5. The ATT is therefore $0.24 * .2 / (.2 + .1 * .4) + 0.3 * 0.1 * .4 / (.2 + .1 * .4) = 0.25$.

Critical values of $\chi^2(df)$			
	Significance level		
df	.10	.05	.01
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21
11	17.28	19.68	24.72
12	18.55	21.03	26.22
13	19.81	22.36	27.69
14	21.06	23.68	29.14
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.81
19	27.20	30.14	36.19
20	28.41	31.41	37.57
21	29.62	32.67	38.93
22	30.81	33.92	40.29
23	32.01	35.17	41.64
24	33.20	36.42	42.98
25	34.38	37.65	44.31
26	35.56	38.89	45.64
27	36.74	40.11	46.96
28	37.92	41.34	48.28
29	39.09	42.56	49.59
30	40.26	43.77	50.89