

Exam 5106/9106 – Fall 2020

1. In a randomized experiment, half of a sample of 1,000 unemployed was randomly offered access to a training program. While some offered access to the training decided not to use it, others initially not offered the training still received it later. After two years their employment status was recorded. These are the data:

Offered	Trained	Obs	Employed	Female	Types in stratum
N	N	400	.8	.4	n(never-takers) & c(ompliers)
N	Y	100	.7	.6	a(lways-takers)
Y	N	100	.9	.4	n
Y	Y	400	.8	.5	a & c

- (a) Compute an estimate of the local average treatment effect (LATE) of training on employment and interpret the parameter.

$$\text{LATE} = .0666667$$

- (b) Compute estimates of the fractions of the population that are “compliers”, “always takers”, and “never takers”.

$$P(c) = .6, P(a) = .2, P(n) = .2$$

- (c) Compute estimates of the fraction female among “compliers”, “always takers”, and “never takers”. Are women more or less likely to respond to the training offer than men?

$$P(\text{female} | a) = 0.6,$$

$$P(\text{female} | n) = 0.4,$$

$$P(\text{female} | c) = (.4 - .25 * .4) / .75 = 0.4 \text{ or } (.5 - .25 * .6) / .75 = 0.46666$$

depending on whether it is computed using the NN or YY stratum

Comparing $P(\text{female} | c)$ to $P(\text{female}) = 0.46$ tells you whether women are more likely to respond to the training offer than men.

- (d) Compute estimates of the mean employment rate in case of training for compliers, the mean employment rate in case of no-training for compliers, the mean employment rate in case of training for “always takers”, and the mean employment rate in case of no-training for “never takers”.

$$E[y_1 | a] = .7$$

$$E[y_0 | n] = .9$$

$$E[y_1 | c] = (.8 - .25 * .7) / .75 = .83333333$$

$$E[y_0 | c] = (.8 - .25 * .9) / .75 = .76666667$$

- (e) What can the data above tell us about the average treatment effect for the treated (ATT)?

The ATT is a weighted average of the LATE + the ATE for always-takers ($P(z=1)=.5$):

$$\begin{aligned} ATT &= E[y1 - y0 \mid c] \frac{.6 \times .5}{.2 + .6 \times .5} + E[y1 - y0 \mid a] \frac{.2}{.2 + .6} \\ &= 0.066667 \times .6 + E[y1 - y0 \mid a] \times .4 \\ &= 0.04 + E[y1 \mid a] \times .4 - E[y0 \mid a] \times .4 \\ &= 0.04 + 0.7 \times 0.4 - E[y0 \mid a] \times .4 \\ &= .32 - E[y0 \mid a] \times .4 \end{aligned}$$

no assumption bounds on $E[y0 \mid a]$ put this in the following range

$$[-.08, .32]$$

2. Schools provide extra tutoring for weaker pupils. You are interested in the average effect of tutoring on exam pass rates.

Consider the following data which reports exam pass rates, sample sizes, separately by treatment status and the average income in the pupils neighborhood (in 3 categories):

	Pupil received:	
	No Tutoring	Tutoring
Neighborhood Income:		
- Low	0.95	0.91
	15,000	5,000
- Middle	0.96	0.95
	15,000	5,000
- High	0.96	0.93
	15,000	5,000
Total	0.96	0.93
	45,000	15,000

- (a) Compute no-assumption bounds on the average treatment effect (ATE).

Note that the potential outcomes are bounded between 0 and 1.

$$E[y_1] = E[y_1 | \text{tutoring}] \cdot p(\text{tutoring}) + E[y_1 | \text{no tutoring}] \cdot (1 - p(\text{tutoring}))$$

$$= 0.93 \cdot 15000/60000 + E[y_1 | \text{no tutoring}] \cdot 45000/60000$$

$$= 0.93 \cdot 0.25 + E[y_1 | \text{no tutoring}] \cdot 0.75$$

$$\in [0.93 \cdot 0.25, 0.93 \cdot 0.25 + 0.75] = [.2325, .9825]$$

similarly

$$E[y_0] = E[y_0 | \text{tutoring}] \cdot p(\text{tutoring}) + E[y_0 | \text{no tutoring}] \cdot (1 - p(\text{tutoring}))$$

$$= E[y_0 | \text{tutoring}] \cdot 0.25 + 0.96 \cdot 0.75$$

$$\in [0.96 \cdot 0.75, 0.25 + 0.96 \cdot 0.75] = [.72, .97]$$

$$\text{and ATE} \in [0.2325 - 0.97, 0.9825 - 0.72] = [-.7375, .2625]$$

- (b) What monotone-treatment assumption (MTS) seems reasonable here? Use potential outcome notation to formulate your assumption and explain your answer.

The tutoring targets the weaker pupils. We would expect these to be weaker in any counterfactual k :

$$E[Y^k | \text{tutoring}] \leq E[Y^k | \text{no-tutoring}]$$

- (c) Use the MTS assumption from (b) to compute MTS-bounds on the ATE.

the MTS gives $E[y_1 | \text{no tutoring}] \in [0.93, 1]$ and $E[y_0 | \text{tutoring}] \in [0, 0.96]$

Since $E[y_1] = 0.93 \cdot 0.25 + E[y_1 | \text{no tutoring}] \cdot 0.75$ we get $E[y_1] \in [0.93, 0.9825]$

Since $E[y_0] = E[y_0 | \text{tutoring}] \cdot 0.25 + 0.96 \cdot 0.75$ we get $E[y_0] \in [0.72, 0.96]$

and therefore $\text{ATE} \in [0.93 - 0.96, 0.9825 - 0.72] = [-0.03, 0.2625]$

- (d) You consider using average income in the neighborhood as a monotone-instrumental variable assumption (MIV). What MIV assumption seems reasonable here? Use potential outcome notation to formulate your assumption and explain your answer.

Neighborhood income correlates with the quality of the home environment, education of the parents, social capital etc. It seems therefore reasonable to assume that on average the higher the income of the the neighborhood (X), the higher the average counterfactual achievement:

$$E[Y^k | X = low] \leq E[Y^k | X = middle] \leq E[Y^k | X = high]$$

- (e) Use the MTS assumption from (b) and the MIV assumption from (d) to compute MTS-MIV-bounds on the ATE.

Using the MTS as in (c) but now conditional on X gives us conditional MTS upper and lower bounds as in the table below. We then use the MIV assumption to tighten these MTS bounds (in bold).

	Tutoring		E[y1 X]				E[y0 X]			
	Yes y0	No y1	MTS LB	UB	+MIV LB	UB	MTS LB	UB	+MIV LB	UB
X:										
- Low	0.95	0.91	0.91	0.9775	0.91	0.9775	0.7125	0.95	0.7125	0.95
- Middle	0.96	0.95	0.95	0.9875	0.95	0.9825	0.72	0.96	0.72	0.96
- High	0.96	0.93	0.93	0.9825	0.95	0.9825	0.72	0.96	0.72	0.96
Total					0.9367	0.9808			0.7175	.9567

This gives $ATE \in [0.9367 - .9567, 0.9808 - 0.7175] = [-0.02, 0.2633]$

3. Assume unemployment duration t_i has the following distribution:

$$f(t; \theta) = (1/\theta) \exp(-t/\theta)$$

with $E[t] = \theta$, $\Pr(t \geq c) = \exp(-c/\theta)$, and $E[t | t \geq c] = \theta + c$.

Durations are right censored at $t = c$ (i.e. we observe $\min(t, c)$) and let d_i denote the censoring indicator (1 if $t_i \geq c$, 0 otherwise).

(a) Write down the log-likelihood for an i.i.d. sample of size n .

$$\mathcal{L} = \sum_{i=1}^n \mathcal{L}_i = - \sum_{i=1}^n (d_i c / \theta + (1 - d_i)(t_i / \theta + \log(\theta)))$$

(b) Derive the score.

$$\frac{\partial \mathcal{L}}{\partial \theta} = \sum_{i=1}^n \frac{\partial \mathcal{L}_i}{\partial \theta} = \sum_{i=1}^n \left(\frac{d_i c + (1 - d_i) t_i}{\theta^2} - \frac{(1 - d_i)}{\theta} \right)$$

(c) Derive the $\hat{\theta}_{MLE}$, the ML estimator of θ .

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0 \Rightarrow \hat{\theta}_{MLE} = \frac{\sum_{i=1}^n d_i c + (1 - d_i) t_i}{\sum_{i=1}^n (1 - d_i)}$$

(d) What is the variance of $\hat{\theta}_{MLE}$?

$$\text{Var}(\hat{\theta}_{MLE}) = -\frac{1}{n} E[\partial^2 \mathcal{L}_i / \partial \theta^2]^{-1} \text{ where}$$

$$\begin{aligned} -E[\partial^2 \mathcal{L}_i / \partial \theta^2] &= E[2(d_i c + (1 - d_i) t_i) / \theta^3 - (1 - d_i) / \theta^2] \\ &= 2cE[d_i / \theta^3] + 2E[t_i / \theta^3] - 2E[d_i t_i / \theta^3] - E[(1 - d_i) / \theta^2] \\ &= 2c \exp(-c/\theta) / \theta^3 + 2/\theta^2 - 2(\theta + c) \exp(-c/\theta) / \theta^3 \\ &\quad - (1 - \exp(-c/\theta)) / \theta^2 \\ &= (1 - \exp(-c/\theta)) / \theta^2 \end{aligned}$$

Suppose $n = 4$, $c = 8$, and t takes on the following values in your sample: 3, 5, 8, 8.

(e) What is the MLE estimate of θ ? What is its standard error?

$$\hat{\theta}_{MLE} = (3 + 5 + 8 + 8) / 2 = 12 \text{ and } s.e. = \frac{12}{2\sqrt{1 - \exp(-8/12)}} = 8.6014623$$

(f) Test for $H_0 : \theta = 8$ against $H_1 : \theta \neq 8$ at the 5% significance level using the LR, Wald and LM test.

$$\begin{aligned} LR &= 2 * (-(3 + 2 * \log(8)) - (3 + 2 * \log(12))) = 1.6218604 \\ W &= (12 - 8)^2 / 8.6014623^2 = 25 / 16 = .2162596 \\ LM &= (1/8)^2 * (8^2 / (4 * (1 - 1/\exp(1)))) = 0.3954942. \\ \text{None is significant at the 5\% level (critical value is 3.84)} \end{aligned}$$

Critical values of $\chi^2(df)$			
	Significance level		
df	.10	.05	.01
1	2.71	3.84	6.63
2	4.61	5.99	9.21
3	6.25	7.81	11.34
4	7.78	9.49	13.28
5	9.24	11.07	15.09
6	10.64	12.59	16.81
7	12.02	14.07	18.48
8	13.36	15.51	20.09
9	14.68	16.92	21.67
10	15.99	18.31	23.21
11	17.28	19.68	24.72
12	18.55	21.03	26.22
13	19.81	22.36	27.69
14	21.06	23.68	29.14
15	22.31	25.00	30.58
16	23.54	26.30	32.00
17	24.77	27.59	33.41
18	25.99	28.87	34.81
19	27.20	30.14	36.19
20	28.41	31.41	37.57
21	29.62	32.67	38.93
22	30.81	33.92	40.29
23	32.01	35.17	41.64
24	33.20	36.42	42.98
25	34.38	37.65	44.31
26	35.56	38.89	45.64
27	36.74	40.11	46.96
28	37.92	41.34	48.28
29	39.09	42.56	49.59
30	40.26	43.77	50.89