University of Oslo / Department of Economics / NCF 2009–03–16

ECON5160: The compulsory term paper

Formalities:

- This term paper is **compulsory**.
- This paper must be accepted in order to qualify for attending the final exam, but will not count on your final grade.
- To be handed in(*): Thursday April 23rd 2009, at 1400 hours at the department office, 12th floor. Do not submit by e-mail unless agreed with the teacher.
- Language: English or Norwegian. Use the attached front page.
- Cooperation allowed, but the paper itself must be in your own words; similar applies for code.
- If a submitted paper is not accepted, you will get a second chance to improve it in a very short timeframe.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Other information:

- There are five problems. Problem 5 is for candidates who have credits in Maths 4 only, and others should not do it.
- In addition to a satisfactory total score based on problems 1–4 (5), you are expected to make a decent attempt on each of them in order to have your paper accepted.
- For problems requiring numerical calculations, you are free to use your favourite software. Your numerical calculations must be documented:
  - If source code is available, attach a print-out
  - Otherwise, write a pseudo-code and attach
  - For some applications where the algorithm is available from a file but not in a source code format, you are welcome to discuss other means of documentation.
- *Keep your files!* In case I would want to put any numerics into the final exam, I do not think we can ask you to do it from scratch. Note that numerics on the final exam is not decided for or against, so this point is just there in order to keep options open.
- I consider this term paper to be more (*much* more) work than e.g. previous Maths 2 or Maths 3 term papers.

Problem 5 (only for those with Math 4 credits):

(a) Show that for two tail-equivalent distributions, with cdfs F and G unbounded to the right, we have that $\mathrm{El}(1-F)/\mathrm{El}(1-G) \to 1$ as x grows, where «El» denotes elasticity.

(b) If $k \ln x + \ln f(x)$ converges as x grows, where f is the pdf of a random variable X, what is the corresponding GEV distribution? (I.e., what is the ξ parameter?)

(c) Let v be a function such that for any $k > 0$, we have $v(kx)/v(x) \to 1$ as x grows. If a cdf F has the property that $(1 - F(x))x^\alpha/v(\alpha x)$ converges to some $c > 0$, what is then the corresponding GEV distribution?
Hint: Test first $G(x) = v(x)x^\alpha$ using a result repeated in the problem set[1], with a linear function a.

(d) Find the ξ value for the GEV corresponding to a Student t distribution with d degrees of freedom.

[1]the original text version erroneously referred to the handouts

Problems for all students (problems 1–4, one page)

Problem 1: Do exercise 10 in the Schweder compendium. Do also simulate the process:

- Fix intensity at 1.
- Let $f(N) = E[T]$ with N as variable. Calculate $R = f(20)$.
- For each N value of 15, 20 and 25, do the following: Graph 10 realizations of X (starting from $X(0) = 1$), time length R, in the same diagram.

Problem 2: Consider exercise 13 in the Schweder compendium, and the sketch in the S_Solutions.pdf file on the course web site.

- Write the solution out in detail, and do the numerical exercise.

Problem 3: Do T&K chapter V problem 6.7.[2]

Problem 4:

(a) As problem (2) in the revised 2009–02–15 note, but with the utility function replaced by $A_t \ln c_t$. Disregard the (P), (N) or boundedness conditions in Seierstad.

(b) Let $dX(t) = mX(t)dt + X(t)dB(t)$ be a geometric Brownian motion, where m is a constant. Consider limits as t grows:

i) For what value(s) of m will $\lim X(t)$ exist but $\lim E[X(t)]$ diverge?

ii) For what value(s) of m will $\lim E[X(t)]$ converge but $\lim X(t)$ diverge?

(Hint: Look up the law of the iterated logarithm, e.g. this Wikipedia article (which shows that in the long run, $B(t)$ grows slower than linearly).)

(c) Interest rate is continuously accumulated at 5% p.a. If an at-the-money[3] European call option maturing time $T = 1$ from now costs 2, what is the Black-Scholes price of an option with the double maturity and strike at 1.2 times today's price? Put today's price = 100. (Hint: Solve numerically.)

[2] The problem originally given in the previous text version of the problem set was 6.8, but 6.7 is a simpler special case. Bonus score for doing 6.8.

[3] i.e.: strike equals today's price