Introduction to Models with Heterogenous Individuals

ECON5160 – Stochastic modeling and analysis

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Overview

1. Can an economic model simultaneously account for income/earnings and wealth inequality?
   More specifically: Can we generate the observed wealth (and consumption) inequality taking the income process exogenously?
2. In order to generate inequality we need idiosyncratic risk + incomplete markets
3. Huggett's endowment economy and Aiyagari's production economy
4. If $u''' \neq 0$: precautionary savings motive
5. That the aggregate equilibrium state variables are constant makes the Huggett-Aiyagari models tractable.

   But is it consistent with data that e.g. unemployment risk and duration of unemployment is uncorrelated with the aggregate state of the economy?

Huggett (1993)

The first model that we will study is Mark Huggett (1993, JEDC). Huggett is interested in a particular issue, the "risk free rate puzzle".

- Proposes a heterogeneous agent model with incomplete markets.
- The only uncertainty is idiosyncratic endowment risk. There is no aggregate uncertainty (e.g., no business cycle).
- If a complete set of state-contingent claims could be traded, then agents would be able to perfectly insure themselves against their idiosyncratic risk.
- If households tend to run up large quantities of savings, the risk free real interest rate will be relatively low compared to a representative agent or complete markets benchmark.

Hugget (1993) cont'd

In particular, Huggett considers an endowment economy with a bond in zero net supply.

- The only asset that can be traded is a one-period un-contingent bond i.e., a claim to one unit of consumption for sure to be delivered in one periods time.
- A borrowing constraint that prevents households issuing too much debt is imposed.
- There are no outside assets so that the net bond position across all households must be zero.

Aiyagari (1994, QJE)

- Arguably the most popular example of a simple incomplete markets model.
- Essentially, Aiyagari presents a production-economy version of Huggett's endowment economy.
- As in Huggett's economy, market incompleteness and idiosyncratic shocks give rise to endogenous heterogeneity and precautionary savings.

Risk free rate puzzle

- Consider the simplest possible response to the equity premium: *maybe people really are really really risk averse*.
- To see why this is not a satisfactory answer, notice that the long run average risk free interest rate will satisfy a steady-state condition like

$$(1+g)^{\sigma} = \beta(1+r)$$

where g is the long run average consumption growth rate, β is the time discount factor, and r is the risk free rate.
- Then we can write

$$r \approx -\ln(\beta) + \sigma g$$

So if you crank up risk aversion, you must also crank up the risk free rate.

Quantitative importance

- In Mehra and Prescott's data, $g = 0.018$ annually.
- Typical estimate of β is about $\beta = 0.96$ annually, so $-\ln(\beta) \approx .04$.
- Suppose then that agents were risk neutral ($\sigma = 0$),
  - then the risk free rate would merely reflect the rate of time preference and $r = 0.04$ or 4% on an annual basis.
- If agents had log preferences ($\sigma = 1$),
  - then the risk free rate would be $r = 0.058$, about 6% on an annual basis.
- And if agents are really really risk averse, say $\sigma = 20$,
  - then the risk free rate would be $r = 0.40$, about 40% on an annual basis!

Hugget (1993) – *an endowment economy*

An individual's endowment y follows a Markov chain with transition probabilities $\pi(y', y)$ where

$$\pi\left(y', y\right) = \Pr\left\{y_{t+1} = y' \mid y_t = y\right\}$$

and where $y, y' \in Y$, a finite set.

Taking as given the bond price q, an individual has the dynamic programming problem

$$V\left(a, y\right) = \max_{a' \geq a} \left\{u\left(c\right) + \beta \sum_{y'} V\left(a', y'\right) \pi\left(y', y\right)\right\}$$

subject to the budget constraint

$$c + qa' \leq a + y$$

Huggett (1993) – Equilibrium

A stationary recursive competitive equilibrium for this economy is: (i) a value function V , (ii) an individual decision rule g, (iii) a stationary probability distribution μ, and (iv) a bond price q s.t.

1. Given the bond price q, the value function V and the individual decision rule g solve the household's dynamic programming problem,
2. The stationary distribution μ is induced via the exogenous Markov chain for y, y' and the decision rule g, and
3. The bond market clears

$$\sum_{a} \sum_{g} g\left(a, y\right) \mu\left(a, y\right) = 0.$$

Aiyagari (1994, QJE) – *Production economy*

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + a_{t+1} = \begin{cases} [1 + (1-\tau) r_t] a_t + (1-\tau) w_t & \text{if } s = e \\ [1 + (1-\tau) r_t] a_t + \omega & \text{if } s = u \end{cases}$$

where c is consumption, a is asset holdings, τ is the tax rate, r is rate of return on assets, w is wage income if employed, ω is unemployment benefits, s is employment status, e stands for employed and u stands for unemployed.

Aiyagari – Markov decision problem

Employment status is governed by a discrete two-state Markov process

$$\pi\left(s' \mid s\right) = \Pr\left\{s_{t+1} = s' \mid s_t = s\right\} = \begin{pmatrix} p_{e,e} & p_{e,u} \\ p_{u,e} & p_{u,u} \end{pmatrix}.$$

The individuals solve their dynamic problem

$$v\left(s,a\right) = \max_{a'}\left\{u\left(c\right) + \beta \sum_{s} \pi\left(s' \mid s\right) v\left(s', a'\right)\right\}$$

Aggregates

Aggregate capital stock is

$$K_t = \sum_{s \in \{e,u\}} \int_{a_{\min}}^{\infty} a_t f(s, a_t)\, da_t,$$

Aggregate labor supply is given by

$$N_t = \int_{a_{\min}}^{\infty} f(e, a_t)\, da_t.$$

Output and factor prices:

$$Y_t = F(K_t, N_t), \quad r_t = F_1(K_t, N_t) \quad \text{and} \quad w_t = F_2(K_t, N_t).$$

Budgets are balanced period-by-period

$$\omega \int_{a_{\min}}^{\infty} f(u, a_t)\, da_t = \tau F(K_t, N_t).$$

Computational procedure

Step 1 Compute the stationary employment N.

Step 2 Make initial guesses of the aggregate capital stock K and the tax rate τ

Step 3 Compute the wage rate w and the interest rate r

Step 4 Compute the households' decision rules.

Step 5 Compute the stationary distribution

Step 6 Compute taxes T and the capital stock

$$K = \sum_{s\in\{e,u\}} \int_{a_{\min}}^{\infty} af(s,a)\, da,$$

Step 7 Compute the tax rate τ that solves the government budget.

Step 8 Update K and τ and return to Step 2 if necessary.

Precautionary saving

As we saw in both

- Huggett (increased demand for bonds, higher equilibrium bond prices, lower return on holding bonds)
- Aiygari (increased savings, lower equilibrium rates of return)

in economies with idiosyncratic shocks and incomplete markets, the demand for insurance in form of savings is higher than in economies with complete markets.

In a partial equilibrium savings problem, it has been known since Leland (1968) and Sandmo (1970) that precautionary savings in response to risk are associated with convexity – or a positive third derivative – of the marginal utility function.

Aggregate uncertainty – Problem

In the economy with only idiosyncratic risk, the distribution of capital is stationary and the integral under the distribution function – i.e. the aggregate capital stock – is constant.

In the economy with aggregate uncertainty, the distribution of capital is *not* stationary. Since individual consumption/savings decisions depend on the factor prices – which are functions of the aggregate capital stock – they need to know *the law of motion for capital*.

But in order to know the law of motion for aggregate capital each individual must know the law of motion for *all moments* of the distribution function of capital. I.e. *the state space explodes* and the problem becomes computationally untractable.

Aggregate uncertainty – the Krusell-Smith algorithm

Individuals only use the first moment of K to predict the law of motion for the capital stock.

E.g. in the case where the aggregate uncertainty can only take two values $Z \in \{Z_g, Z_b\}$

$$\ln K' = \begin{cases} \gamma_{0g} + \gamma_{1g} \ln K & \text{if } Z = Z_g \\ \gamma_{0b} + \gamma_{1b} \ln K & \text{if } Z = Z_b \end{cases}$$

Algorithm: Solve the individuals' problem, simulate the model economy, estimate the l.o.m., solve the individuals' problem ...

Aggregate uncertainty – the Krusell-Smith algorithm – Why??

"Approximate aggregation theorem"
Why?

1. consumption as a function of wealth is concave, but ...
2. most of the savings is done by the ....