ECON5210 Topics in microeconomics - part II: Contract Theory, Spring 2009

Final Exam

Date of Announcement: 30 April, 2009

The deadline for submission is Friday 8 May (or if sent by regular mail, should be poststamped by 7 May).

The answers to the problems must

- either be handed in at the reception on the 12th floor at the Department of Economics before 15:00 on 8 May,
- or sent by ordinary mail with postmark 7 May at the latest; in this case, send the answers to

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- or sent by email as a pdf (preferred) or doc attachment to

tore.nilssen@econ.uio.no
or
tapas.kundu@econ.uio.no
by 15:00 on 8 May at the latest.

Please send questions relating to the understanding of the problems to
tore.nilssen@econ.uio.no or tapas.kundu@econ.uio.no

Question 1

There are two types of consumers for a firm's product, θ_H and θ_L, with respective proportions $1-\lambda$ and λ. A consumer's utility is given by

$$u(x,T) = \theta \frac{1-(1-x)^2}{2} - T,$$

where θ denotes the type, x denotes the amount of the good consumed, and T denotes the payment.

The firm is the sole producer of this good, and its cost of production per unit is $c > 0$.

(i) Consider a nondiscriminating monopolist. Derive his optimal pricing policy. Show that he serves both classes of consumers if either θ_L or λ is "large enough."

(ii) Consider a monopolist who can distinguish the two types (by some characteristic) but can only charge a simple price p_i to each type θ_i. Characterize his optimal prices.

(iii) Suppose the monopolist cannot distinguish the types. Derive the optimal two-part tariff (a pricing policy consisting of a lump-sum charge F plus a linear price per unit purchased of p) under

the assumption that the monopolist serves both types. Interpret. When will the monopolist serve both types?

(iv) Compute the fully optimal nonlinear pricing. How do the quantities purchased by the two types compare with the levels in *(i)* to *(iii)*?

Question 2

Consider the following two-period ($t = 1, 2$) model of implicit incentive. An entrepreneur P owns a machine, and rents it out to a worker A at a rental fee r_t, $t = 1, 2$. Using the machine, A can produce one widget per period at a periodical cost $c_t = b - e_t - \varepsilon_t$, where e_t is the effort (unobserved by P) taken by A in period t; ε_t's are independent random noise (also independent of b), distributed normally with mean 0 and variance σ^2, and b is the machine's unknown cost characteristic. At the beginning of period 1, both P and A do not know the exact value of b, and the common belief about b is that it is normally distributed with mean β and variance ν^2. The effort cost is given by $g(e_t)$, where g is a strictly increasing and strictly concave function. A bears the whole cost, and can sell the widget in a competitive market at a price q.

We assume that both P and A update their beliefs based on the realization of c_1. The periodic rental fee r_t is paid at the beginning of each period, and so similar to the career concern model, the periodic rental is based on the a forecast of the effort decision and the belief on the cost parameter b.

(i) Show that, in the second period, A will exert the optimal effort level in equilibrium.

(ii) Write down the agent's optimization problem to choose the equilibrium first period effort.

(iii) Will A exert more or less or the same effort level in the second period, compared to the first period? Solve for the first period equilibrium effort, and explain.

Question 3

Consider a two-person auction with independent private values distributed uniformly on $[0, 1]$. Consider two alternative assumptions on bidders' utilities: *Case a*: Bidders are risk neutral. In particular, bidder i has net utility $v_i - P$ if she wins the object and has to pay P, and zero utility if she does not win, where $i \in \{1, 2\}$. *Case b*: Bidders are risk averse. In particular, bidder i has utility $\sqrt{v_i - P}$ if she wins the object and has to pay P and zero utility if she does not win.

(i) Find the seller's expected revenue in each of the two cases for the Vickrey (second-price, sealed-bid) auction.

(ii) Do the same for the first-price, sealed-bid auction, assuming a (linear) symmetric bidding equilibrium.

(iii) Discuss.