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Lecture Notes on Incomplete Contracts

Bolton & Dewatripont, ch 11

- Symmetric, but nonverifiable information

- Contract incompleteness

    - Unforeseen contingencies
        - Example: A in Paris is supposed to deliver equipment to B in Oslo.
            - But no-one has anticipated a strike among air-traffic controllers; or
            - an unanticipated change in demand for B's product; or
            - a change in regulations of A's and/or B's businesses; or
            - innovations that make A's products obsolete as input in B's production.

- Simplicity
  - The contract specifies that A sends one item in each delivery, even though B's requirements varies with wear and tear of her existing equipment, because it is too complicated to specify exactly under which conditions more than one item is needed, or those conditions are unknown.
- Limitations
  - The contract is limited to one year, because it is difficult for B to see what her demands will be after that, and it is difficult for A to see what his abilities to deliver will be.

- The consequence of incomplete contracts: *Revisions and renegotiations*.
  - Find a new delivery time given the strike and how the costs of the delay are to be distributed between A and B.
  - Order more items when the contracted delivery is too small and agree on the price of the extra items
  - Extend the duration of the contract and agree on terms for the next period.

- The *ex post* costs of renegotiation
    - Negotiations over revisions take time and resources
    - At the stage of renegotiation, the two parties may be asymmetrically informed
        - B has received information, since the initial contract was written, about the demand for its products. This information is not known to A.

- So why bother to renegotiate?
    - The contract parties cannot switch without costs if they have made <u>*ex-ante* relationship-specific investments</u>.
        - B may have made investments in machinery that is customized to the equipment delivered by A
        - A may have made investments in knowledge about the production process that is specific to the requirements of B.

- The hold-up problem: the *ex ante* cost of renegotiation
    - If such investments in the relationship cause problems for you later on, when revision of the contract is necessary, you may want to invest less than what is socially optimum.
        - Because of the risk of renegotiation later on, A and B use machinery and knowledge that are more general-purpose than what would be profitable for them if a complete contract could be written.
        - If B's machinery can only be used together with equipment from A, then B's bargaining position during renegotiations is weak. B does not want this to happen.

- Institutional design
    - Corporate governance
        - integration or not?
            - hold-up vs incentives when integrated
            - allocation of control rights

The property-rights theory of the firm

- Grossman and Hart (1986), Hart and Moore (1990).

- A set I of agents. A set of physical assets $\overline{A}$. Two stages.
  - o Stage 1: Each agent makes an *ex-ante* investment x_i at a cost $\psi(x_i)$; this investment is potentially specific to some of the assets, $A \subseteq \overline{A}$. Let $x = (x_1, \ldots, x_I)$.
  - o Stage 2: Some subset $S \subseteq I$ of the agents can combine with some of the assets, $A \subseteq \overline{A}$, to generate a surplus from trade, $V(S, A \mid x)$.

- Extreme contractual incompleteness: no initial contract exists.
  - o Only allocation of ownership can take place ex ante. This allocation affects the division of the ex post surplus, not the size of it.

- The *allocation of ownership* is determined by a mapping $\omega(S)$ from the set of all subsets of I to the set of all subsets of $\bar{A}$.
    - The agents in $S \subseteq I$ control the assets in $\omega(S) \subseteq \bar{A}$.
    - An asset can at most be controlled by one of the groups S or its complement $I \backslash S$.

      $$\omega(S) \cap \omega(I \backslash S) = \varnothing$$

    - If an asset is controlled by the subgroup $S' \subseteq S$, then it is also controlled by S.

      $$\omega(S') \subseteq \omega(S).$$

- Multilateral *ex-post* bargaining: How is the surplus distributed among the members of a group?
    - The *Shapley value*: each group member gets the expected contribution of that member to the overall ex post surplus that the trade among group members generates.
- Given an ownership allocation $\omega(S)$, ex ante investments x, and the ex post surplus generated for a given group S of agents, $V(S, \omega(S) \mid x)$, an agent $i \in I$ gets

  $$B_i(\omega \mid x) = \Sigma_{S \mid i \in S}\, p(S)\{ V(S, \omega(S) \mid x) - V(S\backslash\{i\}, \omega(S\backslash\{i\}) \mid x)\},$$

  where $p(S) = \dfrac{(s-1)!(I-s)!}{I!}$

- Example: a printer and a publisher
  - Two agents: $I = 2$. Two assets: $\bar{A} = \{a_1, a_2\}$.
  - Agent 1: printer. Agent 2: publisher.
  - *Ex ante* investments $x = (x_1, x_2)$ at stage 1, trade at stage 2.
  - Three ownership allocations
    - Nonintegration:
      $$\omega(1) = \{a_1\},\ \omega(2) = \{a_2\}.$$
    - Publisher integration:
      $$\omega(1) = \emptyset,\ \omega(2) = \{a_1, a_2\}.$$
    - Printer integration
      $$\omega(1) = \{a_1, a_2\},\ \omega(2) = \emptyset.$$
  - Suppose no *ex-post* surplus can be generated without combining the two assets.

    $V(\{1\}, \{a_1\} \mid x) = V(\{2\}, \{a_2\} \mid x) = 0$

    whereas

    $V(\{1,2\}, \{a_1, a_2\} \mid x) = V(x) > 0$
  - *Nonintegration* (*NI*): Expected payoff to each agent is
    $$B_1(NI \mid x) = B_2(NI \mid x) = \frac{V(x)}{2}.$$

- *Printer integration* (*PI*): It is possible for the printer alone to generate surplus if he owns both assets, but not so much as if he had hired the publisher:

  $V(\{2\}, \varnothing \mid x) = 0$, $V(\{1\}, \{a_1, a_2\} \mid x) = \Phi_1(x_1) < V(x)$

- Expected payoffs to the two agents are not the same, since the printer has a stand-alone value:

$$B_1(PI \mid x) = \frac{V(x) - \Phi_1(x_1)}{2} + \Phi_1(x_1),$$

$$B_2(PI \mid x) = \frac{V(x) - \Phi_1(x_1)}{2}$$

- *Publisher integration* (*pI*) is similar:

$$B_1(pI \mid x) = \frac{V(x) - \Phi_2(x_2)}{2},$$

$$B_2(pI \mid x) = \frac{V(x) - \Phi_2(x_2)}{2} + \Phi_2(x_2).$$

- *Ex ante* investments: Given the ownership allocation $\omega(S)$, each agent chooses his investment x_i according to

$$\frac{\partial}{\partial x_i} B_i(\omega(S) \mid x_1, x_2) = \psi_i'(x_i)$$

  - Investments are not contractible and are chosen non-cooperatively.
  - For example, under nonintegration, equilibrium investments are given by:

$$\frac{1}{2}\frac{\partial}{\partial x_i} V(x_1, x_2) = \psi_i'(x_i),\ i = 1, 2$$

- Printer has greater investment incentives under printer interation than under nonintegration, etc.

- Suppose agents' investments are *complementary*:

$$\frac{\partial^2}{\partial x_1 \partial x_2} V(x_1, x_2) \geq 0.$$

- If $\Phi_i'(x_i) > 0$, then integration provides more investments than nonintegration.
- If $\Phi_i'(x_i) \leq 0$, then nonintegration provides more investments.
  - Example: the investment of agent *i* is fitted to be used together with agent *j*. When he is not there, then the investment is counter-productive.

- The general case
    - Regularity assumptions on V and ψ.
    - Investment complementarity for all pairs of agents i, j.
    - Superadditivity: for all $S' \subseteq S$, $A' \subseteq A$,

      $$V(S, A \mid x) \geq V(S', A' \mid x) + V(S \backslash S', A \backslash A' \mid x)$$

    - Increasing returns to group size and number of assets: for all $S' \subseteq S$, $A' \subseteq A$,

      $$\frac{\partial}{\partial x_i} V(S, A \mid x) \geq \frac{\partial}{\partial x_i} V(S', A' \mid x)$$

    - If marginal returns on investments are higher for all agents under ownership allocation $\hat{\omega}$ than under ω, that is,

      $$\frac{\partial}{\partial x_i} B_i(\hat{\omega} \mid x) \geq \frac{\partial}{\partial x_i} B_i(\omega \mid x), \text{ for all } x \text{ and all } i,$$

      then also equilibrium investment levels are higher:

      $$x_i^e(\hat{\omega}) \geq x_i^e(\omega)$$

- A three-agent example: printer, publisher, bookseller
  - o Suppose again that no ex post surplus can be generated without combining all three assets.
  - o Nonintegration:

$$B_1(NI \mid x) = B_2(NI \mid x) = B_3(NI \mid x) = \frac{V(x)}{3}.$$

  - o Printer integration: Printer also owns publisher asset.

$$B_{1,2}(I \mid x) = B_3(I \mid x) = \frac{V(x)}{2}.$$

  - o By integrating, the printer and the publisher weaken their bargaining position vis a vis the bookseller.
  - o This integration is only profitable if it considerably increases investment levels, so that

$$\frac{V(x_{PI})}{2} - \psi(x_{PI}) > \frac{2V(x_{NI})}{3} - \psi(x_{NI})$$

Supply assurance: substitutable investments

- Three firms: one upstream supplier of inputs to two downstream firms: D_1, D_2, U.
- Only downstream firms make investments: advertising, etc.
- The two firms' investments are substitutable, since they compete for inputs.
  - Increased investment by firm i increases the *ex post* value of the input for this firm, lowers the profit of firm j and therefore discourages firm j from investing.
- Random *ex post* surplus
  - Downstream firm D_i can generate ex post surplus by teaming up with upstream firm U: $V_i(x_i, \theta)$, where x_i is D_i's ex ante investment and θ is a random variable.
  - For some realizations of θ, $V_1 > V_2$; for other realizations, it is the opposite.
- Upstream firm has all bargaining power – sells inputs to highest bidder at a price equal to second-highest valuation $V_i(x_i, \theta)$.
- Efficient investment levels: $E[\frac{\partial}{\partial x_i} V_i(x_i, \theta)] = \psi_i'(x_i)$

- Nonintegration: equilibrium investment levels are efficient.
    - Because of the second-price auction, a downstream firm gets to appropriate its entire ex post rent when it is positive.
- Vertical integration in equilibrium: incentives to *foreclose* the unintegrated downstream firm.
    - Two firms: integrated $\{D_1, U\}$ and unintegrated D_2.
    - The integrated firm appropriates not only all of its own ex post rent but also some of the unintegrated firm's ex post rent.
    - The integrated firm overinvests, the unintegrated firm underinvests.
    - Incentives to integrate: Nonintegration cannot occur in equilibrium.
    - In order to sustain nonintegration, an agreement not to retrade assets is necessary.
        - Difficult to write a complete such contract.

Corporate governance

- Separation ownership/control
- Risk neutral entrepreneur has an idea, but no cash. Needs funds $K > 0$ to start up the firm. Risk neutral investor has funds.
- After start-up, an event θ may occur requiring an action a to be undertaken.
  - Two possible events, good and bad:
    $$\theta \in \{\theta_B, \theta_G\}$$
  - Probability of a good event is $p \in [0, 1]$.
  - Two possible actions, liquidate or continue:
    $$a \in \{a_L, a_C\}$$
  - Private cost or benefit to the entrepreneur from the action:
    $$h(a_j, \theta_i) = h_j^i$$
    - personal satisfaction, reputation, etc.
- When completed, project realizes returns $r \in \{0,1\}$.
  - Probability of high return, $r = 1$, in event θ_i and after action a_j, is y_j^i.
    - y_j^i is also expected return.
- Efficient action: continuation in good event, liquidation in bad.

$$y_C^G + h_C^G > y_L^G + h_L^G, \text{ and } y_C^B + h_C^B < y_L^B + h_L^B$$

- Positive expected return

$$p\, y^G_C + (1-p)\, y^B_L > K.$$

- Incomplete contract: entrepreneur and investor cannot write a contract contingent on the event θ.
- But there is a *signal* – information correlated with the true event – that contracts can be based on.
    - o Signal: $\zeta \in \{\zeta_G, \zeta_B\}$.
    - o Probability of good signal when event is good

      $$p^G = \Pr(\zeta_G \mid \theta_G\} > ½$$

    - o Probability of bad signal when event is bad

      $$p^B = \Pr(\zeta_B \mid \theta_B\} > ½$$

    - o The closer to ½ are p^G and p^B, the more incomplete is the contract. A measure of contract incompleteness:

      $$d = (1 - p^G) + (1 - p^B)$$

    - o Contract: action $a(\zeta)$ and repayment to investor $t(\zeta, r, a)$.
    - o A simpler contract – when action cannot be contract upon (for example because of moral hazard):

      $$t(\zeta, r) = t_\zeta r + k_\zeta, \text{ satisfying: } t(\zeta, r) \leq r.$$

- Entrepreneur control
  - The entrepreneur is interested in both monetary returns and private benefits

    $$a_E^i = \arg\max_j \{ y_j^i (1 - t_l) - k_l + h_j^i \}$$

  - Even if the initial contract is inefficient, for example calling for liquidation when event is good but signal bad, the ex post contract, after renegotiation, is efficient.
    - The investor has funds and is able to bribe the entrepreneur to achieve efficiency.
  - But investor protection is weak – the investor does not always get his share of the returns from renegotiation.
    - Example: Suppose $h_C^G > h_L^G$ and $h_C^B > h_L^B$; the entrepreneur's private benefit is greater with continuation than with liquidation, no matter what the state is.
    - The entrepreneur needs a sufficiently large stake in the firm, or else he will inefficiently choose to continue in the bad state.

- Difference in total payoff in the bad state between the two actions

$$\Delta^B = (y_L^B + h_L^B) - (y_C^B + h_C^B)$$

- Difference in monetary returns

$$\Delta_y^B = y_L^B - y_C^B$$

- Three strategies for the investor and entrepreneur
  1) Rely entirely on renegotiation: Investor gets all monetary returns, but entrepreneur continues even in the bad state. Investor's expected payoff

     $$\Pi_R = p\, y_C^G + (1-p)\, y_C^B$$

  2) Write a *renegotiation-proof contract*: the entrepreneur must now get the proper incentives for choosing liquidation in the bad state

     $$(1 - t_\zeta)\Delta_y^B \geq \Delta_y^B - \Delta^B \Leftrightarrow$$

     $$1 - t_\zeta \geq \frac{\Delta_y^B - \Delta^B}{\Delta_y^B}, \ \zeta = \zeta_G, \zeta_B$$

     Investors' expected payoff:

     $$\Pi_{NR} = [p\, y_C^G + (1-p)\, y_C^B]\, \frac{\Delta^B}{\Delta_y^B}$$

3) Write a *partially renegotiation-proof contract*: a contract that depends on the signal received:

$$t_B = \frac{\Delta^B}{\Delta_y^B},\ t_G = 1,$$

with a corresponding expression for investor's expected payoff, Π_{PR}.

- In sum, entrepreneur control results in efficient investment levels, but investor protection may be so weak that there will be no funding, which happens when

$$\max\{\Pi_{NR}, \Pi_R, \Pi_{PR}\} < K$$

- Investor control
  - The investor always chooses the action with the highest expected monetary return y_j^i. This is not always efficient.
    - If $y_L^G > y_C^G$, then the investor may choose to liquidate in the good state, even though continuation is efficient. – Short-termism.
    - In order to avoid liquidation, the entrepreneur must bribe the investor with an amount $t_\zeta(y_L^G - y_C^G)$.
    - But the entrepreneur has no own funds, and can therefore bribe only with

$$y_C^G(1 - t_\zeta) - k_\zeta.$$

    - *Ex post* efficiency requires

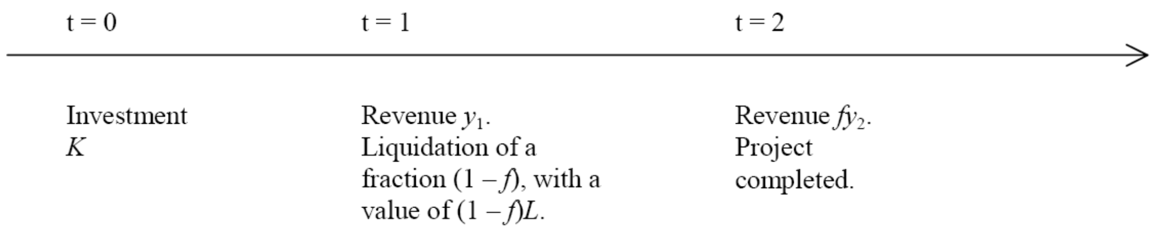
$$t_\zeta \leq \frac{y_C^G - k_\zeta}{y_L^G}$$

    - This may be insufficient to cover investor's expenses, implying that an inefficient contract is necessary
  - In sum, investor protection strong. But efficient action not guaranteed because of entrepreneur's lack of funds to bribe the investor.

- Summary so far
  - Entrepreneur control: always efficient, but may not be feasible
  - Investor control: always feasible, but may not be efficient.
- Contingent control
  - Control to entrepreneur following signal ζ_G, control to investor following signal ζ_B.
  - If entrepreneur control is not feasible and investor control is inefficient, then contingent control may be desirable.
- Empirical studies: venture-capital investment contracts
  - Future financing and control rights are often made contingent on observable measures of firm performance.
  - Kaplan and Stromberg (2003).

Cash flow diversion

- Two periods of returns from project. Entrepreneur may divert cash flow at the early date. Allocation of control may need to depend on whether or not entrepreneur pays back.
- A simple version of Hart and Moore (1998).
- Exposition here based on Hart (1995).



- If the firm does not pay at $t = 1$, then control over the firm is transferred to the creditors and liquidated, in whole or in part.

- Both y_1 and y_2 are non-verifiable.
  - o Correspondance with previous model: fy_2 corresponds to private benefit h, $(1 - f)L$ to verifiable monetary income y.

- Can investor C trust that E does not take the money and run? Can C be sure that the debt is payed down at $t = 1$ when there is revenue available to do it?

- E's initial wealth $w \in (0, K)$.
- Continuing until $t = 2$ is first-best:

$$y_2 > L$$

- Investment is profitable in first best; however, second-hand value of assets is lower than first-hand:

$$y_1 + y_2 > K \geq L$$

- Contract: $\{B, P\}$
  - E borrows $B \geq K - w$ at $t = 0$ and promises to pay $P \leq L$ at $t = 1$.
  - If he defaults on his debt, C seizes control and liquidate the project.
- Scope for *renegotiation*: In case E defaults, it is possible for him to renegotiate with C in order to carry on part of the project.
- Entrepreneur control: E has all bargaining power.
- $t = 2$: E keeps everything, fy_2.

- $t = 1$: E prefers to give C as much as possible, since $y_2 > L$.
  - E has available

    $$w + B - K + y_1.$$

  - Thus, f is determined as:

    $f = 1$, if $w + B - K + y_1 \geq P$

    f solves:

    $$w + B - K + y_1 + (1 - f)L = P,$$

    otherwise.

- E has all bargaining power $\Rightarrow P = B$

$$\Rightarrow \quad f = \min\left[1, 1 - \frac{K - w - y_1}{L}\right] \qquad (\gamma)$$

- $t = 0$: Will E obtain financing?
  - Yes, if the contract is
    - credible: $P \leq L$
    - sufficient: $B \geq K - w$

      implying: $L \geq K - w$; liquidation value greater than capital need

    and
    - E must prefer these conditions to not investing. This is fine if $f = 1$. If $f < 1$, then it is OK if:

      $$fy_2 \geq w$$

      Inserting for w from (γ):

      $$y_1 + fy_2 + (1 - f)L \geq K$$

- In summary,
  - funding may not be feasible

    $$y_1 + fy_2 + (1 - f)L < K$$
  - outcome may not be efficient

    $$f < 1$$