Topics in Microeconomics ECON 5210 (Part II - Contract Theory)

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Introduction

- Strategic interaction between privately informed agents
  - Inadequacy of general equilibrium set up

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- Strategic interaction between privately informed agents
  - Inadequacy of general equilibrium set up
- Private information on
  - on what the agent does (hidden actions)
  - on who the agent is (hidden information)
- Information and sequence of actions
  - Adverse Selection models :
    - Screening - The un-informed party (UP), who is imperfectly informed about the informed party (IP), moves first
    - Signaling - Same informational situation as before, but IP moves first
  - Moral Hazard models : UP, who is imperfectly informed about IP's action, moves first

Introduction

Overview of the course

- Bilateral contracting with private information (class 1 & 2)
  - Adverse selection
  - Moral hazard
  - Multidimensional informational problems
- Contracting under multilateral asymmetry (class 3)
- Repeated Contracting (class 4 & 5)
  - Dynamic moral hazard, Dynamic adverse selection
- Incomplete contracts (class 6)

Adverse Selection

Bilateral setting with discrete types

- B buys a good from S
- B gets $\theta v(q) - T$
- S gets $\pi = T - cq$
- Assume $\theta = \begin{cases} \theta_H \text{ with probability } 1-\beta \\ \theta_L \text{ with probability } \beta \end{cases}$ $\theta_L < \theta_H$
- Assume that S sets the terms of contract; B has reservation utility $\overline{u}$.

Bilateral setting with discrete types

First-best Outcome

S knows B's type

S solves

$$\max_{T_i, q_i} T_i - cq_i$$

subject to $\theta_i v(q_i) - T_i \geq \overline{u} \quad (IR)$

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B receives reservation utility $\rightarrow \theta_i v(q_i) - T_i = \overline{u}$

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B receives reservation utility $\rightarrow \theta_i v(q_i) - T_i = \overline{u}$

Optimal q_i solves $\theta_i v'(q_i) = c$.

Bilateral setting with discrete types

Second-best Outcome: Non linear pricing

S cannot observe B's type, so S has to offer a set of choices $T(q)$, independent of B's type

The optimization problem

$$\max_{T_i, q_i} \beta \left[T(q_L) - c q_L \right] + (1 - \beta) \left[T(q_H) - c q_H \right]$$

subject to

$$\theta_i v(q_i) - T_i > 0 \quad (IR)$$

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subject to

$$\theta_i v(q_i) - T_i > 0 \quad (IR)$$

and

$$q_i = \arg\max_q \theta_i v(q) - T(q)$$

Revelation Principle

Without loss of generality, can restrict ourselves to the pair of optimal choices made by two types of buyers $\{[T(q_L), q_L], [T(q_H), q_H]\}$
However, they have to be incentive compatible.

$$\begin{aligned} \theta_H v(q_H) - T(q_H) &\geq \theta_H v(q_L) - T(q_L) \\ &\quad \textit{and} \\ \theta_L v(q_L) - T(q_L) &\geq \theta_L v(q_H) - T(q_H) \end{aligned}$$

The optimization problem

$$\max_{T_i, q_i} \beta \left[T(q_L) - cq_L \right] + (1-\beta) \left[T(q_H) - cq_H \right]$$

subject to

$$IRL : \theta_L v(q_L) - T_L \geq 0$$

$$IRH : \theta_H v(q_H) - T_H \geq 0$$

$$ICL : \theta_L v(q_L) - T(q_L) \geq \theta_L v(q_H) - T(q_H)$$

$$ICH : \theta_H v(q_H) - T(q_H) \geq \theta_H v(q_L) - T(q_L)$$

Solving the model

- *IRL* is binding

If not, increase $T(q_L)$ and $T(q_H)$ by the same amount

$$\theta_H v(q_H) - T(q_H) > \theta_H v(q_L) - T(q_L) > \theta_L v(q_L) - T(q_L)$$

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$$\theta_H v(q_H) - T(q_H) > \theta_H v(q_L) - T(q_L) > \theta_L v(q_L) - T(q_L) = 0$$

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$$\theta_H v(q_H) - T(q_H) > \theta_H v(q_L) - T(q_L) > \theta_L v(q_L) - T(q_L) = 0$$

- $q_H > q_L$ (add *IC*s)
- *ICL* and *IRH* can be ignored.$(ICH + \{q_H > q_L\} \rightarrow ICL)$

Solving the model

$$\max_{q_H, q_L} \beta \left[\underbrace{\left(\theta_L v\left(q_L\right) - c q_L\right) - \frac{1-\beta}{\beta}\left(\theta_H - \theta_L\right) v\left(q_L\right)}_{\text{virtual surplus from type } L} \right] + (1-\beta)\left[\theta_H v\left(q_H\right) - c q_H\right]$$

$$\theta_H v'\left(q_H\right) = c$$

$$\theta_L v'\left(q_L\right) = \frac{c}{1 - \left(\frac{1-\beta}{\beta} \frac{\theta_H - \theta_L}{\theta_L}\right)} > c$$

If S increases q_L, it makes the $\left[T\left(q_L\right), q_L\right]$ package more alluring to type H. To prevent type H from choosing $\left[T\left(q_L\right), q_L\right]$, he must therefore reduces $T\left(q_H\right)$

Positive surplus for type H

Optimal allocation

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Optimal allocation

- Every type but the highest gets sub-efficient allocation
- Every type but the lowest receives a positive informational rent. An agent of type H gets an informational rent as he can always pretend his type as L
- Every type but the lowest is indifferent between his contract and that of immediately lower type

Application

- Credit market: A bank facing borrowers with different risk profile (Stiglitz and Weiss 81)
- Insurance market: An insurer providing insurance for agents with different levels of risk (Stiglitz 77)
- Trade off between allocative efficiency and redistributive taxation (Mirrless 86)
- Regulating a natural monopolist(Baron and Myerson 82, Laffont and Tirole 86)

Regulating a natural monopolist - Laffont Tirole 82

Think about an indivisible project run by a firm for the government.

Firm's cost $c = \theta - e$; c is observable, but not θ or e

costly effort $\psi(e) = \frac{e^2}{2}$

Given a subsidy of amount s, the firm gets $s - c - \psi(e)$

Government wants to have the project done at minimum cost

First best: If it had known θ, $e = \theta - c$ will be observable. Then the government can reimburse the effort cost $(s = \psi(e))$, and asks the firm to produce effort e^* that minimizes total cost

$$\begin{aligned} e^* &= \arg\max_e s + c = \arg\max_e \theta - e + \psi(e) \\ &\Rightarrow \psi'(e^*) = 1 \end{aligned}$$

Effort level is independent of type, but actual cost is higher for inefficient type.

Regulation under Adverse Selection

Suppose $Pr(\theta = \theta_L) = \beta$

Government offers a cost-contingent contract $\{[s_L, c_L], [s_H, c_H]\}$

$$\min \left[\beta (s_L - e_L) + (1 - \beta)(s_H - e_H)\right]$$

$$s_L - \psi(e_L) \geq 0$$

$$s_H - \psi(e_H) \geq 0$$

$$s_L - \psi(e_L) \geq s_H - \psi(e_H - \Delta\theta)$$

$$s_H - \psi(e_H) \geq s_L - \psi(e_L + \Delta\theta)$$

Regulation under Adverse Selection

Note that θ_H is the inefficient type here.

IR is binding for θ_H

IC is binding for θ_L

$$\min \left(1-\beta\right)\left(\psi\left(e_H\right)-e_H\right)+\beta\left(\psi\left(e_L\right)-e_L+\psi\left(e_H\right)-\psi\left(e_H-\Delta\theta\right)\right)$$

$$\psi'\left(e_L\right)=1; \psi'\left(e_H\right)-\left(1-\beta\right)\psi'\left(e_H-\Delta\theta\right)-\beta=0$$

No distortion in e_L; Allocative efficiency for the efficient type, but comes at an informational rent

But, $e_H < e^*$

Stochastic Contract

- B faces a lottery instead of a fixed allocation
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  - Since *IRL* is binding, and if θ_L is risk averse, S can only charge something less than $T_L^{**} < T_L^*$ given a lottery over q with mean q_L^*.
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    - *ICH* is binding for the deterministic contract. As θ_L is getting a worse deal than $[T_L^*, q_L^*]$, S can possibly charge a little more from θ_H
- Gain over type H exceeds loss over type L if type H is more risk averse (Maskin and Riley 84)

Continuous types

$$max_{q(\theta),T(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [T(\theta) - cq(\theta)] dF(\theta)$$

subject to

$$IR : \theta v(q(\theta)) - T(\theta) \geq 0$$

$$IC : \theta v(q(\theta)) - T(\theta) \geq \theta v\left(q\left(\widehat{\theta}\right)\right) - T\left(\widehat{\theta}\right)$$

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What about IC constraints?

Given the Spence-Mirrless Condition, which is going to be satisfied in our scenario,

$$\frac{\partial^2 u}{\partial\theta\partial q} \text{ is of constant sign } (> 0)$$

where $u(\theta, q, T) = \theta v(q) - T$;

ICs are equivalent of

$Monotonicity : \frac{dq(\theta)}{d\theta} \geq 0$

$Local\ IC : \theta v'(q(\theta)) \frac{dq(\theta)}{d\theta} = T'(\theta)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$

The Optimization Problem

$max_{q(\theta),T(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [T(\theta) - cq(\theta)] dF(\theta)$

subject to

$IR : \underline{\theta} v(q(\underline{\theta})) - T(\underline{\theta}) \geq 0$

$Monotonicity : \frac{dq(\theta)}{d\theta} \geq 0$

$LIC : \theta v'(q(\theta)) \frac{dq(\theta)}{d\theta} = T'(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$

Mirrless' approach

Define the rent for type θ : $W(\theta) = \theta v(q(\theta)) - T(\theta)$

As q and T will be chosen optimally, we have $\frac{dW(\theta)}{d\theta} = v(q(\theta))$ or
$W(\theta) = \int_{\underline{\theta}}^{\theta} v(q(x))\, dx$
(notice that I am using LICs here)

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Next, no incentive to give the lowest type any positive surplus
$\underline{\theta} v(q(\underline{\theta})) - T(\underline{\theta}) = 0$ (all IRs are satisfied)

$\max_{q(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q(\theta)) - \left[\int_{\underline{\theta}}^{\theta} v(q(x))\, dx\right] - cq(\theta)] dF(\theta)$

First order condition

Integrate by parts, and then look at the first order condition

$$\left[\theta - \frac{1-F(\theta)}{f(\theta)}\right] v'\left(q\left(\theta\right)\right) = c$$

Suboptimal allocation for all types other than the highest type.

Let $q^*\left(\theta\right)$ be the solution of the above problem.

First order condition

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Suboptimal allocation for all types other than the highest type.

Let $q^*(\theta)$ be the solution of the above problem.

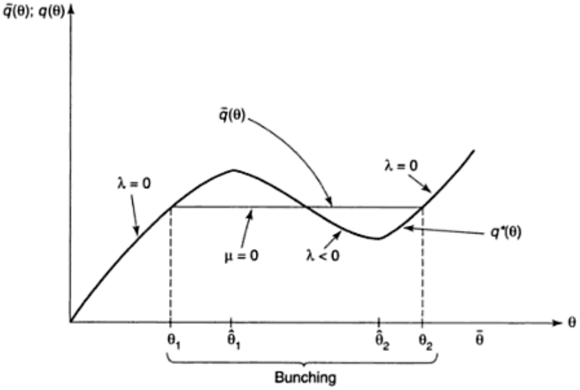
But does it satisfy $\frac{dq^*(\theta)}{d\theta} \geq 0$?

It turns out that a sufficient condition is the hazard rate $\frac{f(\theta)}{1-F(\theta)}$ to be increasing in θ

Bunching

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} [\theta v(q(\theta)) - \left[\int_{\underline{\theta}}^{\theta} v(q(x))\,dx\right] - cq(\theta)]dF(\theta)$$

subject to *Monotonicity* : $\frac{dq(\theta)}{d\theta} \geq 0$. Let $\overline{q}(\theta)$ be the solution.



Adverse selection with multidimensional types

Consider a seller with some market power who sells at least two different goods. Example, a supermarket.

Bundling can be a screening device (Adams and Yallen 76)

State of Nature	v_1	v_2	Prob
A	90	10	$\frac{1}{4}$
B	80	40	$\frac{1}{4}$
C	40	80	$\frac{1}{4}$
D	10	90	$\frac{1}{4}$

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Without bundling, optimal price charged : $P_1 = P_2 = 80$
Expected profit $= \frac{80}{2} + \frac{80}{2} = 80$

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Without bundling, optimal price charged : $P_1 = P_2 = 80$
Expected profit $= \frac{80}{2} + \frac{80}{2} = 80$
Alternate strategy: sell good 1 and good 2 each at price $P_1 = P_2 = 90$ and offer the bundle at $p_b = 120$

When is bundling optimal?

McAfee, McMillan, and Whinston 89

A monopolist selling two different items, 1 and 2

Buyers' valuation $(v_1, v_2) \in [\underline{v}_1, \overline{v}_1] \times [\underline{v}_2, \overline{v}_2]$ distributed as $F(v_1, v_2)$

Cumulative Distribution (and density): $H_i(v_i)$ [and $h_i(v_i)$]

Cost of production: c_1, c_2

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As buyer can buy the goods separately, we have $P_b < P_1 + P_2$

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As buyer can buy the goods separately, we have $P_b < P_1 + P_2$

Let P_1^* and P_2^* be the monopoly prices.

Consider an alternate offer: $P_1 = P_1^*, P_2 = P_2^* + \varepsilon, P_b = P_1^* + P_2^*$

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Consuming 1 only: $v_1 \geq P_1^*$ and $v_2 \leq P_2^*$

Consider an alternate offer: $P_1 = P_1^*, P_2 = P_2^* + \varepsilon, P_b = P_1^* + P_2^*$

Consuming 1 only: $v_1 \geq P_1^*$ and $v_2 \leq P_2^*$

Consuming 2 only: $v_2 \geq P_2^* + \varepsilon$ and $v_1 \leq P_1^* - \varepsilon$

Consider an alternate offer: $P_1 = P_1^*, P_2 = P_2^* + \varepsilon, P_b = P_1^* + P_2^*$

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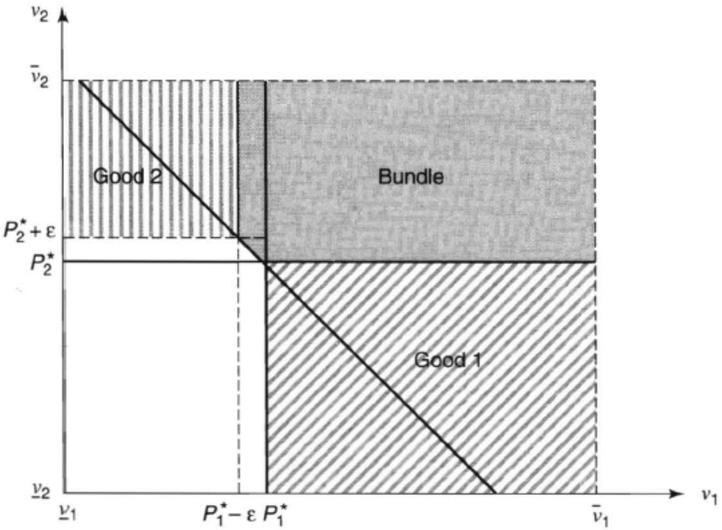
Consuming 2 only: $v_2 \geq P_2^* + \varepsilon$ and $v_1 \leq P_1^* - \varepsilon$

Consuming the bundle: $v_1 + v_2 \geq P_1^* + P_2^*, v_2 \geq P_2^*$ and $v_1 \geq P_1^* - \varepsilon$


v_2
$\bar{v}_2$
Good 2
Bundle
$P_2^* + \varepsilon$
P_2^*
Good 1
$\underline{v}_2$
$\underline{v}_1$
$P_1^* - \varepsilon$ P_1^*
$\bar{v}_1$
v_1

Local incentive

Find whether the seller has incentive to increase P_2 from P_2^*

$$\pi(\varepsilon) = (P_2^* + \varepsilon - c_2) \int_{v_1}^{P_1^* - \varepsilon} \left[\int_{P_2^* + \varepsilon}^{\overline{v}_2} f(v_1, v_2)\, dv_2 \right] dv_1$$
$$+ (P_1^* + P_2^* + c_1 - c_2) \int_{P_1^* - \varepsilon}^{P_1^*} \left[\int_{P_2^* + P_1^* - v_1}^{\overline{v}_2} f(v_1, v_2)\, dv_2 \right] dv_1$$

If we assume valuations are drawn independently, then the first order condition can be written as

$H_1(P_1^*) \{[1 - H_2(P_2^*)] - (P_2^* - c_2) h_2(P_2^*)\}$
$+ (P_1^* - c_1)[1 - H_2(P_2^*)] h_1(P_1^*)$, which is always positive

If not independent, bundling is not always optimal.

Armstrong and Rochet 99 provides a rigorous global analysis.

References and exercises

- Adverse Selection
  - Discrete types (BD Ch. 2.1)
  - Examples including regulation (BD Ch. 2.2)
  - Stochastic Contracts (BD Ch. 2.3.2)
  - Continuous types (BD Ch. 2.3.3)
  - Multidimensional types (Bundling) (BD Ch. 6.1.1.,6.1.2)
- Problems BD 2.2, BD 2.3