Topics in Microeconomics ECON 5210 (Part II - Contract Theory)

Tapas Kundu & Tore Nilssen

University of Oslo

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General optimal contract

P maximizes $V(q-w)$

A's utility is given by $u(w) - \psi(a)$

Usual structural assumptions on V, u and ψ

$\max_{\{w(q),a\}} \int_{\underline{q}}^{\overline{q}} V(q-w) f(q \mid a)\, dq$

subject to

$IR: \int_{\underline{q}}^{\overline{q}} u(w(q)) f(q \mid a)\, dq - \psi(a) \geq \overline{u}$

$IC: a \in \arg\max_{\widehat{a}} \left\{ \int_{\underline{q}}^{\overline{q}} u(w(q)) f(q \mid \widehat{a})\, dq - \psi(\widehat{a}) \right\}$

A's choice problem

$$a \in \arg\max_{\widehat{a}} \left\{ \int_{\underline{q}}^{\overline{q}} u\left(w\left(q\right)\right) f\left(q \mid \widehat{a}\right) dq - \psi\left(\widehat{a}\right) \right\}$$

The objective function is not necessarily concave

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If we replace *IC* by the first order and the second order conditions

$$FOC_A : \int_{\underline{q}}^{\overline{q}} u\left(w\left(q\right)\right) f_a\left(q \mid a\right) dq = \psi'\left(a\right)$$

$$SOC_A : \int_{\underline{q}}^{\overline{q}} u\left(w\left(q\right)\right) f_{aa}\left(q \mid a\right) dq - \psi''\left(a\right) < 0$$

Consider the constrained problem with only the FOC_A

A's choice problem

Deviation from Borch rule

$\frac{V'[q-w(q)]}{u'(q)} = \lambda + \mu \frac{f_a(q|a)}{f(q|a)}$

deviation from the first best coinsurance scheme as long as $\mu > 0$

Condition that ensures $\mu > 0$:risk averse agent and $F_a(q \mid a) \leq 0$ (FOSD)

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Implication of $\mu > 0$:if q with $f_a\,(q \mid a) > 0$ getting higher reward compared to the first best coinsurance

Validity of the first order approach

But FOC_A and $FOCD$ do not necessarily make the first order approach valid

We also need $MLRP : \frac{d}{dq}\left[\frac{f_a(q|a)}{f(q|a)}\right] \geq 0$ and $CDFC : F_{aa}(q \mid a) \geq 0$

Intuition: $MLRP$ makes the wage schedule monotone (increasing), which in turn makes agent's problem a concave one

$\max \int_{\underline{q}}^{\overline{q}} u(w(q)) f(q \mid a) \, dq - \psi(a)$

(integrating by parts and then look at the second order effect with respect to action)

Moral hazard and renegotiation

If effort is sunk, can we renegotiate the risk sharing after the effort is incurred and sunk?

Is such renegotiation is allowed, the optimal post-effort scheme would be to split the surplus at the first best coinsurance scheme

However, anticipation of such possibilities going to reduce the effort incentive on the part of the agent

What if P anticipates a mixed action?

Non-observable effort

Fudenberg and Tirole 90

Timing: P offers an initial menu of contract C_0 based on performance (success H, failure L)

If A rejects, the game ends. Otherwise, A takes action $(a = 1)$ incurring cost ψ with some probability x

Action is sunk, once taken, A has private information, (but P can only anticipate the probability in equilibrium)

P screens by offering $\{[w_L(0), w_H(0)], [w_L(1), w_H(1)]\}$; A can accept either of the two, or opt for C_0

Can restrict attention to contracts $C_0 = \{[w_L(0), w_H(0)], [w_L(1), w_H(1)]\}$, which are renegotiation-proof

Non-observable effort

Renegotiation-proof contracts

Renegotiation problem when given x

$$\min x[p_1 \widehat{w}_H(1) + (1-p_1)\widehat{w}_L(1)] + (1-x)[p_0 \widehat{w}_H(0) + (1-p_0)\widehat{w}_L(0)]$$

subject to

$$IRL: p_1 u(\widehat{w}_H(1)) + (1-p_1) u(\widehat{w}_L(1)) \geq p_1 u(w_H(1)) + (1-p_1) u(w_L(1))$$

$$IRH: p_0 u(\widehat{w}_H(0)) + (1-p_0) u(\widehat{w}_L(0)) \geq p_0 u(w_H(0)) + (1-p_0) u(w_L(0))$$

$$ICL: p_1 u(\widehat{w}_H(1)) + (1-p_1) u(\widehat{w}_L(1)) \geq p_1 u(\widehat{w}_H(0)) + (1-p_1) u(\widehat{w}_L(0))$$

$$ICH: p_0 u(\widehat{w}_H(0)) + (1-p_0) u(\widehat{w}_L(0)) \geq p_0 u(\widehat{w}_H(1)) + (1-p_0) u(\widehat{w}_L(1))$$

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$IIC : u(w^*) = p_0 u(w_H(1)) + (1 - p_0) u(w_L(1))$

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First stage problem: Solve for x with two additional constraints IIC and

$AIC : u(w^*) = p_1 u(w_H(1)) + (1 - p_1) u(w_L(1)) - \psi$ (indifference)

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$Results$: x should not be too high

FT 90 shows that dx/dw^* may be positive if and only if the agent has decreasing absolute risk aversion

Implicit incentive

Holmström 82: Career concern

What if agents are exposed to risk in terms of their payment

2 periods: $q_t = \theta + a_t + \varepsilon_t$

P offers fixed wage contract, $w_1, w_2(q_1)$:no explicit incentive

A maximizes sum of two period surplus:$w_1 + w_2(q_1) - \psi(a_1) - \psi(a_2)$

Easy to see that $a_2 = 0$ and a_1 maximizes $Ew_2(q_1) - \psi(a_1)$

$$w_2(q_1) = E(\theta, q_1 \mid a_1) = \int \theta \frac{f(\theta, q|a_1)}{\widehat{f}(\theta|a)} d\theta$$

A chooses a_1 to maximize $E[E(\theta, q_1 \mid a_1)] - \psi(a_1)$

$$FOC : \int \int \theta f(\theta, q \mid a_1^*) \frac{\widehat{f}_a(q|a_1)}{\widehat{f}(\theta|a)} dq d\theta = \psi'(a_1^*)$$

$$Cov\left(\theta, \frac{\widehat{f}_a}{\widehat{f}}\right) = \psi'(a_1^*)$$

$H82$ considers $\theta \sim N\left(\overline{\theta}, \sigma_\theta^2\right)$ and $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$; With these distributions $q_1 \sim N\left(\overline{\theta} + a_1^*, \sigma_\theta^2 + \sigma_\varepsilon^2\right)$ so that $\frac{\widehat{f}_a(q|a_1)}{\widehat{f}(\theta|a)} = \frac{\theta - \overline{\theta} + \varepsilon}{\sigma_\theta^2 + \sigma_\varepsilon^2}$

$$Cov\left(\theta, \frac{\widehat{f}_a}{\widehat{f}}\right) = \frac{var(\theta)}{\sigma_\theta^2 + \sigma_\varepsilon^2} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}$$

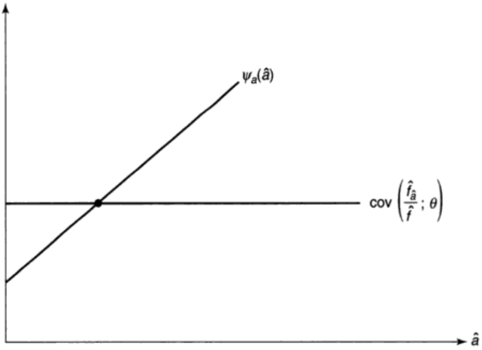

Figure 10.3
Additive Case ($\mu = 0$)

References and exercises

- Moral hazard: First order approach (BD Ch. 4.4)
- Combining moral hazard and adverse selection (BD Ch. 6.3)
- Dynamic Moral hazard
  - Renegotiation with non-observable effort (BD Ch. 10.3.1)
  - Implicit incentive (BD Ch. 10.5)
  - Intertemporal risk sharing (BD Ch. 10.1)