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Lecture Notes on Multilateral Contracting

Bolton & Dewatripont, ch 7

More than one agents have private information

- Bilateral trading
    - both seller and buyer has private information
- Auctions
    - several buyers with private information

Bilateral trading

- Suppose first only buyer has private information
  - Seller (uninformed party) having all the bargaining power: optimum offer may include cutoff – not *ex-post* efficient.
  - Buyer (informed party) having all the bargaining power: always efficient, since buyer knows seller's valuation.

- With one-sided private information, *bargaining power should be with the informed party*.
  - If uninformed has bargaining power, he will seek to reduce the informed party's information rent, at the cost of *ex-post* efficiency.

- Bilateral asymmetric information
  - Both the buyer *and* seller has private information about own valuation.
  - The difficulty is obtaining both incentive compatibility *and* participation. In general, there is no way to always obtain efficient trade.

Auctions

- One seller, multiple buyers, each with private information about her valuation

- Equivalently: one buyer, multiple sellers – *contract auction*.

- Perfectly known values
    - Each buyer has perfect information about her valuation, and valuations are independent across buyers.

- Imperfectly known common values
    - The true valuation is the same for all buyers, but each buyer has her own estimate of this value.

Auctions with perfectly known values

- A simple example
  - One seller. Two risk neutral buyers.
  - A buyer's valuation is
    - $v_i = v_H$ with probability β_i,
    - $v_i = v_L$ with probability $(1 - \beta_i)$
  - $v_H > v_L \geq 0$
  - β_i is the seller's prior belief that buyer *i* has high valuation v_H.
  - Symmetry: Let $\beta_1 = \beta_2 = \beta$.
  - Seller's valuation: 0; seller's cost.
  - Seller sets terms of trade.
  - Single buyer: seller sets price at v_H if $\beta \geq v_L/v_H$; monopoly pricing.
  - The optimal auction is a *mechanism* that specifies, for each pair of buyers' reported types, the price to be paid, and who gets the good; formally,
    - Payment from buyers to seller in the four states: $\{P_{HH}, P_{HL}, P_{LH}, P_{LL}\}$
    - Contingent trades: $\{x_{HH}, x_{HL}, x_{LH}, x_{LL}\}$, where x_{ij} is the probability of buyer *i* getting the good when reported types are v_i and v_j.

- The following constraints need to be satisfied
  - Feasibility

    $2x_{HH} \le 1;\ 2x_{LL} \le 1;\ x_{HL} + x_{LH} \le 1.$

  - Buyer participation
    - high type (IRH)

$$\beta(x_{HH}v_H - P_{HH}) + (1-\beta)(x_{HL}v_H - P_{HL}) \ge 0$$

    - low type (IRL)

$$\beta(x_{LH}v_L - P_{LH}) + (1-\beta)(x_{LL}v_L - P_{LL}) \ge 0$$

  - Buyer incentive compatibility
    - high type (ICH)

$$\beta(x_{HH}v_H - P_{HH}) + (1-\beta)(x_{HL}v_H - P_{HL}) \ge \beta(x_{LH}v_H - P_{LH}) + (1-\beta)(x_{LL}v_H - P_{LL})$$

    - low type (ICL)

$$\beta(x_{LH}v_L - P_{LH}) + (1-\beta)(x_{LL}v_L - P_{LL}) \ge \beta(x_{HH}v_L - P_{HH}) + (1-\beta)(x_{HL}v_L - P_{HL})$$

- As is standard, (IRL) and (ICH) bind, while (IRH) and (ICL) are redundant.

- An auction is *efficient* if the item is always allocated to the highest-valuation user.
    - In our simple example, efficiency requires
$$x_{HL} = 1 \text{ and } x_{LH} = 0$$
    - In addition, when we restrict attention to symmetric auctions, where identical buyers are treated identically, efficiency also requires
$$x_{HH} = x_{LL} = ½$$
- Restrict first attention to efficient auctions. What is the optimum mechanism for the seller among efficient auctions?
    - The *x*s given, so we only need to specify the *P*s.
    - Useful notation: expected payments, for each type
$$P_L^e = \beta P_{LH} + (1 - \beta)P_{LL}$$
$$P_H^e = \beta P_{HH} + (1 - \beta)P_{HL}$$
    - It is sufficient to find P_L^e and P_H^e, because of risk neutrality.

- Seller's problem:

$\max 2[\beta P_H^e + (1-\beta) P_L^e]$

subject to

$\beta v_H/2 + (1-\beta)v_H - P_H^e \geq (1-\beta)v_H/2 - P_L^e$ (ICH)

$(1-\beta)v_L/2 - P_L^e \geq 0$ (IRL)

- Solution:

$$P_L^e = \frac{(1-\beta)v_L}{2}, \quad P_H^e = \frac{v_H}{2} + \frac{(1-\beta)v_L}{2}$$

- Can the seller do even better if he is *not restricted to efficent auctions*?
  - Note similarity with the monopolist's problem: cutting off the low-valuation buyer increases the chance the good is unsold, but also increases the price obtained given that it is sold.

- Now, we need to reintroduce the *x*s.

- Seller's problem:

$$\max 2[\beta P_H^e + (1-\beta) P_L^e]$$

subject to

$$[\beta x_{HH} + (1-\beta)x_{HL}]v_H - P_H^e = [\beta x_{LH} + (1-\beta)x_{LL}]v_H - P_L^e \qquad \text{(ICH)}$$

$$[\beta x_{LH} + (1-\beta)x_{LL}]v_L = P_L^e \qquad \text{(IRL)}$$

$$2x_{HH} \le 1,\ 2x_{LL} \le 1,\ x_{HL} + x_{LH} \le 1 \qquad \text{(feasibility)}$$

  - Solve for P_H^e and P_L^e from (ICH) and (IRL) and insert in objective function to find an expression which
    - is increasing in x_{HH}, so $x_{HH} = ½$ in optimum.
    - is increasing in x_{HL}, so $x_{HL} = 1 - x_{LH}$ in optimum.
    - is increasing in $[x_{HL} - x_{LH}]$, so $x_{HL} = 1$ and $x_{LH} = 0$ in optimum
    - is increasing in x_{LL} if and only if

      $$\frac{\beta}{1-\beta} \ge \frac{v_H - v_L}{v_L},$$

      so $x_{LL} = ½$ in optimum if this holds, else $x_{LL} = 0$.

- The optimum auction may not be efficient
  - Efficiency at the top
  - When both buyers have low valuation, the good may not be traded.
    - The optimum auction improves on the best efficient one when both buyers have low valuation and the probability that this should happen is sufficiently high.
  - *Reservation price* if low valuation sufficiently likely.
    - Providing a high-valuation buyer with incentives to bid high when chances are great that the other buyer has low valuation.

<u>Standard auctions</u>

- Open bid vs sealed bid
- Open bids
  - Ascending bids – English auction
  - Descending bids – Dutch auction
- Sealed bids
  - First-price
  - Second-price – Vickrey auction

<u>Revenue equivalence</u>

- Assume: risk neutrality, symmetry, independence.

- All four auctions give seller the same expected revenue

- English auction and Vickrey auction give same equilibrum outcome.

- First-price sealed-bid auction and Dutch auction are strategically equivalent.

- English auction and Vickrey auction
    - English
        - Dominant strategy for bidder *i* to continue bidding as long as current bid is lower than v_i.
        - Efficient if no reservation price.
        - The price is (just above) second highest valuation.

    - Vickrey
        - Dominant strategy for a bidder *i* to bid equal to her valuation v_i.
            - Let

              bidder's valuation = v_i

              bidder's bid = b_i

              largest bid from others = a
            - What is optimum bid? Distinguish between two cases:

              $a > v_i$: bidder's decision does not matter

              $a < v_i$: bidder wins if $b_i > a$, and earns $v_i - a$
            - Bidding $b_i < v_i$ reduces bidder's chance to win but does not affect what she has to pay if he wins. Optimum to bid $b_i = v_i$.
        - Efficient if no reservation price.
        - The price equals second highest valuation.

- Expected revenue to seller from English and Vickrey auctions in the simple example: Price is high if both have high valuation, otherwise low.

$$\beta^2 v_H + (1 - \beta^2) v_L$$

- Dutch auction and first-price sealed bid auction
  - Strategy sets are the same in the two auctions
    - Dutch auction: a strategy for a bidder is which price to accept, given one's valuation.

  - Allocation rule is the same: Good goes to highest bidder at highest bid.
  - No dominant strategies for players.
  - No pure-strategy equilibrium in the simple example, because of discrete distribution of valuations.
    - In equilibrium, a low-valuation bidder bids $b(v_L) = v_L$.
    - The high-valuation bidder *shades* her bid
      - A bid $b < v_H$ decreases her chances of winning, but also decreases the price *if* she wins.
      - Since a bid slightly above v_L has a probability $(1 - \beta)$ of winning, earning her $(v_H - v_L)$, she will never bid higher than b', where

$$v_H - b' = (1 - \beta)(v_H - v_L) \Leftrightarrow$$

$$b' = \beta v_H + (1 - \beta)v_L$$

  - Mixed strategy with support $f(b|v_H)$ on (v_L, b')
- Expected payoff to bidders is the same in Dutch and first-price sealed-bid auctions as in English and Vickrey auctions: low-value 0, high value $(1 - \beta)(v_H - v_L)$.
- Therefore also expected revenue for the seller is the same

- <u>Revenue equivalence</u>
  - All four auctions provide seller with the same expected revenue: $\beta^2 v_H + (1 - \beta^2)v_L$
  - Less than the optimal efficient auction, since here high-valuation bidder's incentive constraint is not binding
  - This difference disappears when reservation price is allowed.

Continuous types

- An example of bidding in a first-price sealed-bid auction
  - Two buyers. Valuations uniformly distributed on [0,1].
  - Let $g_i(b_i)$ be buyer i's subjective probability that she wins with a bid b_i.
  - If she has valuation θ_i, then she solves

    $$\max\ (\theta_i - b_i)g_i(b_i)$$

    - Trading off the probability of winning with the payoff upon winning.
  - First-order condition:

    $$(\theta_i - b_i)g_i'(b_i) - g_i(b_i) = 0$$

  - Looking for a symmetric equilibrium with bids increasing in valuation: $b_i = b(\theta_i)$ such that $db/d\theta_i > 0$.
    - Implying: $g(b(\theta_i)) = \theta_i$.
  - Rewriting first-order condition:

    $$[g(b_i) - b_i]g'(b_i) - g(b_i) = 0 \Leftrightarrow$$

    $$g(b_i) = 2b_i \Leftrightarrow b_i = \theta_i/2.$$

  - Each bidder bids half her valuation.

Auctions with correlated values: Surplus extraction

- Two types. Two bidders.
- Let β_{ij} be the probability that bidder 1 has valuation i and bidder 2 has valuation j, $i, j \in \{H, L\}$.
- Correlation: $\xi := \dfrac{\beta_{HH}\beta_{LL}}{\beta_{HL}\beta_{LH}} \neq 1$.
- Standard auctions: seller's expected revenue as in the uncorrelated case: $\beta_{HH}v_H + (1 - \beta_{HH})v_L$.
- But the optimal auction takes advantage of the correlation and extracts all the surplus from the bidders.
  - Back to the general contract:
    - Payment P_{ij} from bidder 1 when her valuation is i and bidder 2's valuation is j.
    - Efficient allocation: $x_{HH} = x_{LL} = ½$, $x_{HL} = 1$, $x_{LH} = 0$.
- A contract extracts all surplus if it is incentive compatible at the same time as participation constraints are binding for both types.
- Both participation constraint binding:

$$\beta_{LL}\left(\frac{v_L}{2} - P_{LL}\right) - \beta_{LH}P_{LH} = 0 \qquad \text{(IRL)}$$

$$\beta_{HH}\left(\frac{v_H}{2} - P_{HH}\right) + \beta_{HL}(v_H - P_{HL}) = 0 \qquad \text{(IRH)}$$

- Rewrite participation constraints:

$$P_{LH} = \frac{\beta_{LL}}{\beta_{LH}}(\frac{v_L}{2} - P_{LL}) \qquad \text{(IRL)}$$

$$P_{HL} = v_H + \frac{\beta_{HH}}{\beta_{HL}}(\frac{v_H}{2} - P_{HH}) \qquad \text{(IRH)}$$

- Incentive constraints:

$$\beta_{LL}(\frac{v_L}{2} - P_{LL}) - \beta_{LH}P_{LH} \geq$$

$$\beta_{LL}(v_L - P_{HL}) + \beta_{LH}(\frac{v_L}{2} - P_{HH}) \qquad \text{(ICL)}$$

$$\beta_{HH}(\frac{v_H}{2} - P_{HH}) + \beta_{HL}(v_H - P_{HL}) \geq$$

$$-\beta_{HH}P_{LH} + \beta_{HL}(\frac{v_H}{2} - P_{LL}) \qquad \text{(ICH)}$$

- Insert binding participation constraints into incentive constraints:

$$\frac{\beta_{LH}}{2}(v_L - \beta v_H) - \beta_{LL}(v_H - v_L) + \beta_{LH}(\xi - 1)P_{HH} \leq 0 \qquad \text{(ICL)}$$

$$\frac{\beta_{HL}}{2}(v_H - \beta v_L) + \beta_{HL}(\xi - 1)P_{LL} \leq 0 \qquad \text{(ICH)}$$

- Correlation makes it possible to satisfy both ICs

    - Positive correlation: $\xi > 1$.
        - Put P_{LL} and P_{HH} low enough.
            - The high type is kept from disguising as low type by a low P_{LL}, implying – by (IRL) – a high P_{LH}.
            - Because of positive correlation, the chances are high that also the other buyer is high type, implying that a high-type buyer disguising as low type will have to pay P_{LH}

    - Negative correlation: $\xi < 1$.
        - Put P_{LL} and P_{HH} high enough.

- Note: assumptions of risk neutrality and deep pockets are crucial. Extreme values of payments as $\xi \to 1$.

Risk averse bidders

- Bidding in English and Vickrey auction not affected by risk aversion.

- In Dutch and first-price sealed-bid auctinns, risk aversion leads to more aggressive bidding.

    - A bidder balances the probability of winning against the payoff in case of a win. An increase in the bid
        - increases the chance of winning and therefore getting something, but
        - lowers the gain in case of winning

    - A risk averse bidder is more concerned about the first affect and therefore bids more aggressively than an risk neutral would.

- First-price sealed bid auction: Risk neutral bidders.
  - Two types, two bidders. Low-value bidder always bids $b(v_L) = v_L$ (whether risk neutral or risk averse).
  - High-value bidder bids according to the cumulative distribution function $F(b)$ such that, in a symmetric equilibrium, we have two expressions for expected payoff when her bid is b:

$$[(1-\beta)+\beta F(b)](v_H-b)=(1-\beta)(v_H-v_L)$$

$$\Leftrightarrow F(b)=\frac{1-\beta}{\beta}\left[\frac{v_H-v_L}{v_H-b}-1\right]$$

- Risk averse bidders: vN-M utility function $u(\cdot)$, $u' > 0$, $u'' < 0$.
  - High-value bidder bids according to the cdf $\tilde{F}(b)$ such that:

$$[(1-\beta)+\beta\tilde{F}(b)]u(v_H-b)=(1-\beta)u(v_H-v_L)$$

$$\Leftrightarrow \tilde{F}(b)=\frac{1-\beta}{\beta}\left[\frac{u(v_H-v_L)}{u(v_H-b)}-1\right]$$

- Since $u'' < 0$, we have:

$$\frac{u(v_H-v_L)}{u(v_H-b)}<\frac{v_H-v_L}{v_H-b}\Leftrightarrow \tilde{F}(b)<F(b)$$

  - The probability of a high bid from a risk averse high-valuation bidder is higher than from a risk neutral one.

Observable differences among bidders

- Example: Contract auctions with some foreign and some domestic firms.
- The simple example: two types, two bidders.
- Asymmetry: $\beta_1 < \beta_2$.
  - Bidder 2 has a higher probability of having high valuation.
  - Bidder 1 is weaker than bidder 2.
- Again, English and Vickrey auctions work as before.
- In a first-price sealed-bid auction, the weak bidder bids more aggressively.
- The optimal auction in the general case (multiple types): *favoritism*. Favoring weak bidders in order to induce higher bids from strong bidders.
    - For example: McAfee &McMillan, "Government procurement and international trade", *J Internat Econ* 1989.

Auctions with imperfectly known common values

- Uncertainty about own valuation adds nothing extra, if bidder valuations are independent and bidders are risk neutral.
- Correlated values. Example: objects that can be resold – art, treasury bills. Others' valuations unknown, and own valuation depends on resale value.
- Simple example: True value $v \in \{H, L\}$, $H > L > 0$. Prior belief: $\Pr(H) = ½$.
- Signal – an independent private estimate of the value: $s_i \in \{s_H, s_L\}$.
  - Probability of receiving high (low) signal when value is high (low): $p > ½$.
  - Buyer's expected value upon receiving signal:

$$v_H = pH + (1 - p)L$$
$$v_L = pL + (1 - p)H$$

- Winner's curse
  - With imperfectly known value, the bidding process in itself may provide information to bidders that may induce them to revise their valuations.
  - In particular, winning tells a bidder that she received an estimate that was higher than all other estimates.
  - Consider a Vickrey auction in the simple example. Suppose that bids depend on information: $b(s_H) > b(s_L)$. Winning, and the price you have to pay, tell you what signal the other bidder received, and you revise beliefs accordingly.
    - If both bidders bid high, then

$$\Pr(H|\, s_H, s_H) = \frac{\Pr(H)\Pr(s_H, s_H \mid H)}{\Pr(s_H, s_H)} =$$

$$\frac{\frac{1}{2}p^2}{\frac{1}{2}p^2 + \frac{1}{2}(1-p^2)} = \frac{p^2}{p^2 + (1-p^2)} > p$$

and the valuation is

$$E[v \mid s_H, s_H] = \frac{p^2 H + (1-p^2)L}{p^2 + (1-p^2)}$$

$$> pH + (1-p)L = v_H$$

– an instance of "winner's blessing"

- But if both bidders bid low, then similarly

$$E[v \mid s_L, s_L] = \frac{p^2 L + (1 - p^2)H}{p^2 + (1 - p^2)}$$

$$< pL + (1 - p)H = v_L,$$

"winner's curse" – upon winning when you submitted the low bid, you realize that you were in fact too optimistic.

o Bidders take the risk of the winner's curse into account when bidding:

$$b_H = E[v \mid s_H, s_H];\ b_L = E[v \mid s_L, s_L]$$

- Because the bidding procedure discloses more information, bidders in an English auctions tend to bid more aggressively than in, say, a first-price sealed-bid auction.
- Optimal auction with imperfectly known common value

    o Surplus extraction.

Auctions – topics not covered

- Asymmetric information among bidders
  - Porter, *Econometrica* 1995.
- Multi-unit auctions: quotas, securities
- Combined bids: royalty + fee; vague projects: price + content
- Entry costs, number of bidders, participation fee
- Auctioning incentive contracts
- Repeated auctions
  - ratchet effects
  - capacity constraints
  - collusion among bidders