Exercise: Asset demand and asset pricing

5th November 2009

We start by looking at an individual consumer who lives for two periods (period 0 and period 1). She can invest in two risky assets. She maximizes expected utility

$$E(U) = E[u(C_0) + \frac{1}{1+\rho}u(C_1)] \tag{1}$$

She starts period 0 with initial assets worth A_0 in terms of consumption goods. Labor income in two periods are Y_0 and Y_1 respectively. Y_1 is stochastic. At the end of period 0 the consumer invests the amounts A_1 and A_2 in the two assets. Their (stochastic) rates of return in period 1 are respectively r_1 and r_2.

(1) Write down the period budget constraints for the consumer for both periods.

(2) Derive the first-order conditions for the optimal choice of A_1, A_2, C_1 and C_2. Rewrite them by exploiting that $E(x\dot{y}) = ExEy + Cov(x, y)$. Interpret the conditions. Describe what they mean for the allocation of savings between the two assets.

(3) Suppose the return on asset one is risk-free. How do you interpret the first-order condition for the risk-free asset now. What is now the condition for an optimal allocation of savings between the two assets?

(4) Retain the assumption that r_1 is risk-free. Suppose that the period utility function is quadratic:

$$u(C) = C - \frac{a}{2}C^2, \qquad a > 0 \tag{2}$$

Show that the demand for asset no 2 (the risky asset) can be expressed as

$$A_2 = -\frac{Cov(Y_1, 1+r_2)}{Var(1+r_2)} + \frac{E[u'(C_1)]}{aVar(1+r_2)}[E(1+r_2) - (1+r_1)] \tag{3}$$

Explain in words what this means. How do you interpret the coefficient in front of the rate of return difference?

(5) We now return to the case where both assets are risky. Suppose there is a finite number of states of the world. Under what conditions will it be possible for the consumer to insure fully against income risk?

(6) Suppose there are two states of the world, s_1 and s_2 which are equally probable. Outcomes are described by

	s_1	s_2
Y_1	1.5	2.5
$1+r_1$	1.0	1.0
$1+r_2$	0.8	1.2

Furthermore $\rho = 0$, $A_0 = 0$, $Y_0 = 1.5$ and $a = 0.2$. The utility function is quadratic. Find the asset demands. Give an intuitive explanation for the result.

(7) Leave the numerical example above aside. Suppose the economy is inhabited by a number of identical consumers. Their labor incomes are perfectly correlated. The economy can transfer resources from period 0 to period 1 either by storing the output, or by sowing it. The first method yields a gross return of 1.0 (no shrinkage). The second method yields a gross return of 0.5 in dry years and 1.5 in wet years. Dry an wet years have equal probability. The two assets correspond to to the two different methods of transferring resources. Write down the equilibrium conditions for this economy. What can you say about equilibrium expected returns and about the use of the two methods for storing resources?

(8) At last, let there be two different types of consumers, A and B. They differ only with respect to the level of income risk. Expected incomes are the same for both of them. For the economy as a whole there is no possibility for transferring resources between the two periods. There is only one risk free asset. Characterize the equilibrium in this case. Will there be any trade in the asset? Does the income risk play any role in the determination of the interest rate?

	s_1	s_2	s_3
Y_A	1.0	2.0	3.0
Y_B	2.0	1.0	3.0

Your are asked to suggest how the allocation you got with only a risk-free asset. How many would you suggest and what would they look like?

In the seminar we shall also go through the remaining exercises from last time