Problem Set on RBC

1. **Intertemporal Labor Supply**

Consider the following problem

$$\max_{\{c_t,h_t,A_t\}_{t=0}^{T}} E\sum_{t=0}^{T}\beta^t\left\{\log(c_t)+\psi\frac{(1-h_t)^{1-\sigma}-1}{1-\sigma}\right\}$$
$$s.t.$$
$$A_{t+1} = R(A_t+w_th_t-c_t),\ A_0 \ given$$

where c_t is consumption, h_t is labor supply, A_0 is initial wealth, $R = 1+r$ is the gross rate of return for saving. Here only w_t, the wage rate, is stochastic.

(a) Derive and interpret the first order conditions for c_t, h_t and A_{t+1}. What are the intertemporal and intratemporal optimality conditions?

(b) Judging from the first order conditions, which variables link c_t and h_t in our model? What is the economic interpretation of this variable?

(c) Define the Frisch elasticity of labor supply as

$$\eta = \frac{\partial \ln h_t}{\partial \ln w_t} \mid u'(c) \text{ constant}$$

That is, Frisch elasticity of labor supply capture the elasticity of hours worked to wage rate, given the marginal utility of consumption unchanged. In other words, Frisch elasticity measures the substitution effects of a change in wage rate on labor supply. (Why is that? Find out what marginal utility of consumption in the first order condition is equal to and you will get the answer.) Compute η in this model.

(d) Use the Focs and discuss the effect of the following situations on c_t, and h_t.

i. Unexpected temporary shocks to wages

ii. Unexpected permanent shocks to wages

2. **Calibration**

Consider the following version of the real business cycle model. The economy is populated by a large number of identical agents with total measure one. Each agents has a time endowment of 1 per period. Agents rank life time consumption and leisure according to

$$E\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t},h_{t}\right),\ \beta\in(0,1)$$

There is a large number of identical firms with aggregate technology given by

$$y_t = e^{z_t}F\left(k_t,h_t\right)$$

where z_t is a discrete state Markov process with transaction probability matrix Π and stationary distribution P. The function F is assumed to be strictly increasing in each of its arguments, strictly concave, constant returns to scale and it also satisfies the INADA conditions. The law of motion for the aggregate capital stock is given by

$$k_{t+1} = (1-\delta)\,k_t + i_t$$

where δ is the depreciation rate of capital. The resource constraint is given by $c_t + i_t = y_t$

(a) Write the Bellman's equation for the social planner's problem.

(b) Now assume that

$$\begin{aligned} u\left(c_t,h_t\right) &= \log\left(c_t\right)-\psi h_t \\ F\left(k_t,h_t\right) &= k_t^{\alpha}h_t^{1-\alpha} \end{aligned}$$

Compute the deterministic steady-state for the social planner's problem. A satisfactory answer should consist of three equations that can be used to solve for steady state value of k, c, h.

(c) Calibrate this model economy, so that its competitive allocation matches the long run observations

- $c/y = 0.85$
- $k/y = 3.0$,

- labor share of income is equal to 70%,
- and the fraction of leisure in total nonsleeping hours is equal to 80%.

In particular you need to choose values for α, δ, β, ψ to match the above targets.

3. **Balanced Growth Path**
Consider the following version of the representative agent economy. The stand-in household maximize

$$\sum_{t=0}^{\infty} \beta^t N_t \log c_t$$

where N_t is the number of members in the household. Each member of the household is endowed with an initial capital k_0 at the beginning of time 0, and 1 unit of labor for each $t \geq 0$. Population grow at a constant rate so that $N_{t+1} = \eta N_t$. There are two possible technologies in this economy:

$$\begin{aligned} T_1 &: \quad Y_t = \gamma^t K_t^{\alpha} N_t^{1-\alpha} \\ T_2 &: \quad Y_t = \gamma^t K_t^{\mu} N_t^{\phi} L_t^{1-\mu-\phi} \end{aligned}$$

In both cases, total factor productivity grows at a rate $\gamma > 1$. Also, K_t, Y_t and L_t denote total capital stock, output and land at time t, respectively. Land is a non-reproducible factor and also it does not depreciate. Let's assume that $L_t = 1$ for all t. Assume full depreciation of capital, the resource constraint is given by

$$N_t c_t + K_{t+1} = Y_t$$

(a) Assume that only the first technology is available to the representative firm.

   i. Formulate the social planner's problem as a dynamic programming problem

   ii. Solve for the balanced growth path of this economy. Solve explicitly for the growth rate of per capita consumption c_t along this path. (Hint, the problem requires you to derive the first order conditions and obtained the balanced growth rate from these conditions)

(b) Repeat part (a) using the second technology in place of the first.