Business Cycle Facts and Standard RBC Theory

Advanced Macroeconomic Theory

What are business cycles

- Burns and Mitchell (1913, 1927, 1946): expansions occurring at the same time in many economic sectors, followed by similarly general recessions, contractions and revivals.
  - fluctuations occurs in aggregate activity, not in particular sectors.
  - cycles are recurrent, but not periodic.
  - cycles have at least two different stages: expansions and contractions.
  - once the economy enters into one of the stages, it stays there for some time.

- there are regular and predictable co-movements between variables over the cycles.

- Lucas (1981): deviations of real aggregate output from a trend, where there is co-movement of deviations from trend of various economic aggregates with those of output.

- NBER: a recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.

Questions on Business Cycles

- What are the empirical characteristics of business cycles?
- What brings about business cycles? What propagates them?
- Who is mostly affected and how larger would be the welfare gains of eliminating business cycles?
- What is Great Depression from a Neoclassical Perspective?

Approach and Structure of Lecture

- Document stylized facts of business cycles.
- Construct a theoretical business cycle model to explain business cycles
- Evaluate ability of models to generate business cycles of realistic magnitude
- Use the business cycle model to diagnose Great Depression

1 Data: Decomposition of Growth and Cycles

Question: how to decompose the time series into a long-run trend component and a business cycle component?

- What is growth component and what is cyclical component?

Hodrick-Prescott filter

- want to decompose the raw data, $\log(Y_t)$ into a growth trend $y_t^{trend} = \log\left(Y_t^{trend}\right)$ and a cyclical component $y_t = \log\left(Y_t^{cycle}\right)$ such that

$$\begin{aligned} \log(Y_t) &= \log\left(Y_t^{trend}\right) + \log\left(Y_t^{cycle}\right) \\ y_t &= \log(Y_t) - y_t^{trend} \end{aligned}$$

- The HP-filter proposes to make this decomposition by solving the following minimization problem

$$\min_{y_t, y_t^{trend}} \sum_{t=1}^{T} (y_t)^2 + \lambda \sum_{t=1}^{T} \left[\left(y_{t+1}^{trend} - y_t^{trend} \right) - \left(y_t^{trend} - y_{t-1}^{trend} \right) \right]^2$$

subject to

$$y_t + y_t^{trend} = \log (Y_t)$$

- Trade-off in choosing the trend:
  - want the trend component to be a smooth function by its definition
  - want to make the trend component track the actual data to some degree, in order to capture also some fluctuations in the data that are of lower frequency than business cycles.

- These two considerations are balanced by the parameter λ

- If λ is big, we want to make $\sum_{t=1}^{T}\left[\left(y_{t+1}^{trend}-y_{t}^{trend}\right)-\left(y_{t}^{trend}-y_{t-1}^{trend}\right)\right]^{2}$, the change in the growth rate of the trend component, small.

- High λ makes it optimal to have a trend component with fairly constant slope. In the extreme, as $\lambda \to \infty$, it is optimal to set this term to 0 for all time period, that is

$$y_{t+1}^{trend}-y_{t}^{trend}=g=y_{t}^{trend}-y_{t-1}^{trend}$$

  which is a constant linear trend.

- Note that in high frequency, it is not likely that the trend growth with change.

- If $\lambda = 0$, then the objective function becomes

$$\min_{y_t, y_t^{trend}} \sum_{t=1}^{T} \left(\log(Y_t) - y_t^{trend} \right)^2$$

subject to

$$y_t + y_t^{trend} = \log(Y_t)$$

- The solution is $y_t^{trend} = \log(Y_t)$, and $y_t = 0$.
  - The trend is equal to the actual data and the business cycle components are zero.

- We want to pick a $\lambda \in (0, \infty)$.

- Which λ to choose must be guided our objective of filtering out business cycle fluctuations.

- Which λ accomplishes this depends crucially on the frequency of the data: for quarterly data, a value of $\lambda = 1600$ is commonly used, which loads into the trend component fluctuations that occur at frequencies of roughly eight years or longer.

  – for annual data, $\lambda = 100$ is commonly used.

2 Business Cycle Facts

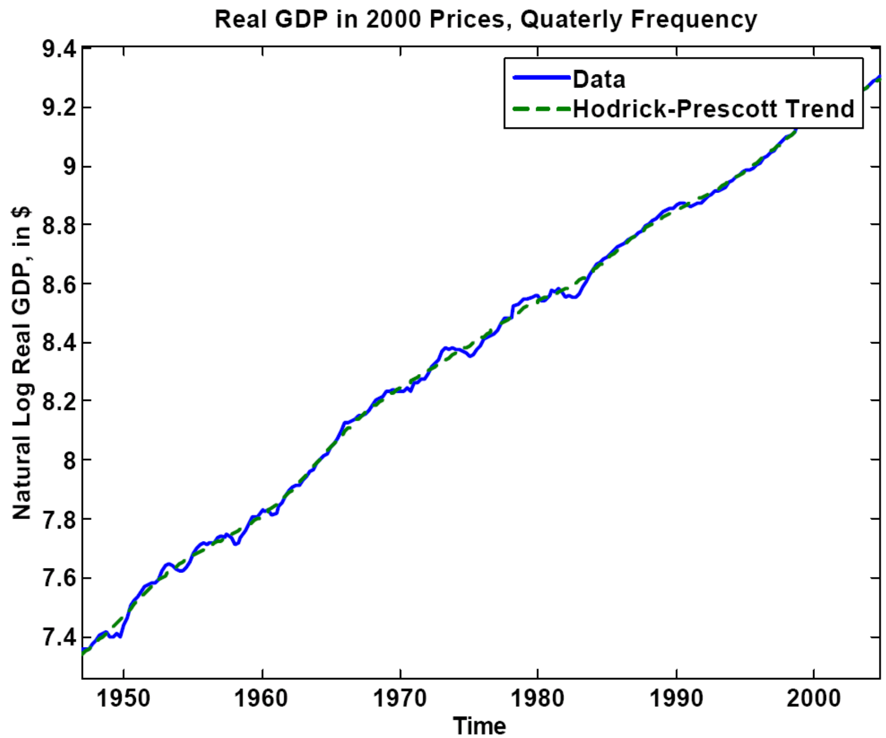


Figure 2.4: Trend Component of HP-Filtered Real GDP for the US, 1947-2004

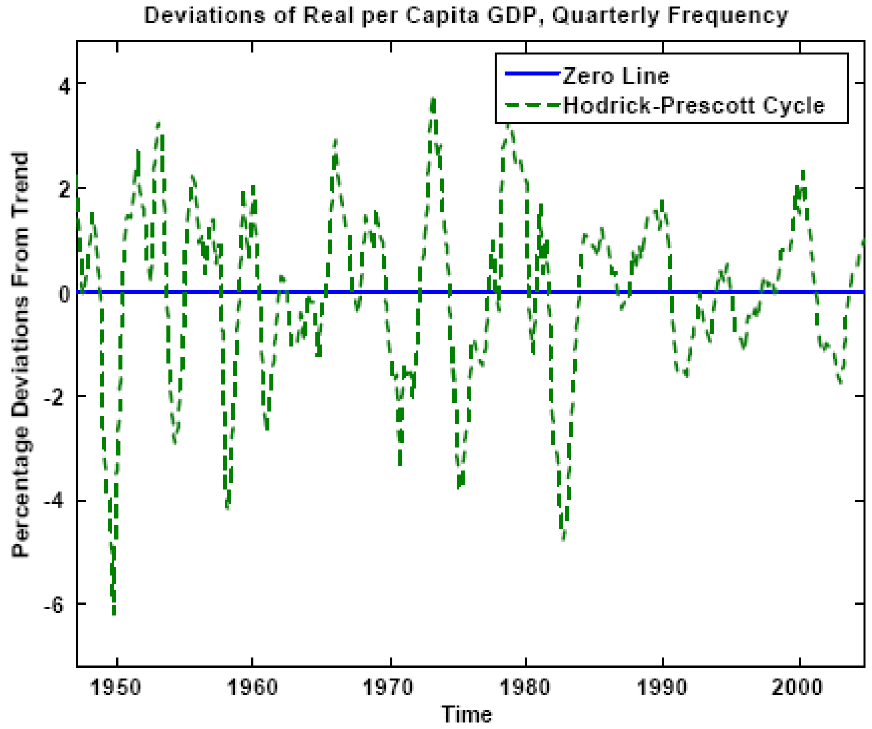


Figure 2.5: Cyclical Component of HP-Filtered Real GDP for the US, 1947-2004

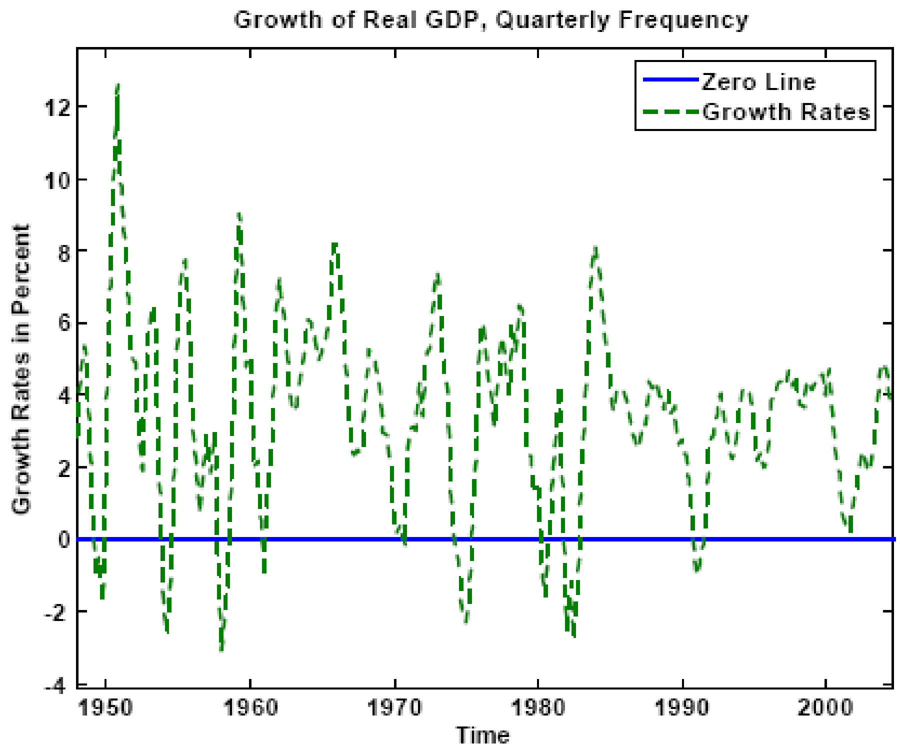


Figure 2.3: Growth Rates of Real GDP for the US, 1947-2004

Statistics of Real GDP Growth Rate

Growth Rate of real GDP	Mean	St. Dv.	min	max	% (> 0)
Raw	3.3%	2.6%	−3.1%	12.6%	88.6%
HP filtered	0%	1.7%	−6.2%	3.8%	53.9%

Autocorrelation of Real GDP Growth Rate

Growth Rate of real GDP	A(1)	A(2)	A(3)	A(4)	A(5)
Raw	0.83	0.54	0.21	-0.09	-0.20
HP filtered	0.84	0.60	0.32	0.08	-0.10

Summary

- Real GDP has a volatility of 1.7% around trend when considering the HP-filtered series.
- Cyclical component of real GDP is highly persistent (positive deviations are followed with high likelihood of positive deviations). Autocorrelation declines with the order and turns negative for the fifth order.
- Positive deviation from trend are more likely than negative deviations from trend. This suggests that recessions are short and sharp; expansions are long and gradual.
- It is rare that the growth rate of real GDP actually becomes negative.

For other macro variables, study the following properties

- Procyclical, countercyclical or acyclical
- Leading or lagging
- More or less variable than the output (volatility)

Some definitions

- Procyclical: a variable that usually increase in booms, decreases in recession. For example, productivity is procyclical.

- Countercyclical: a variable that usually decrease in booms, increases in recession. For example, unemployment.

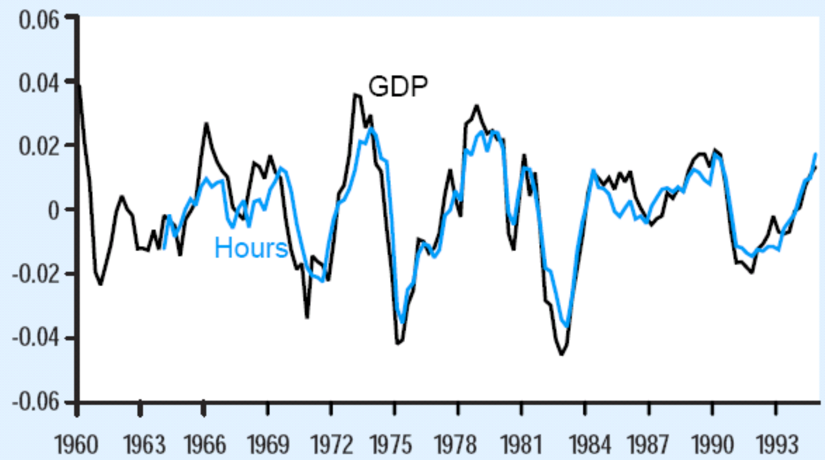
- Acyclical: a variable that shows no systematic relationship to the business cycle.

2.1 Stylized facts

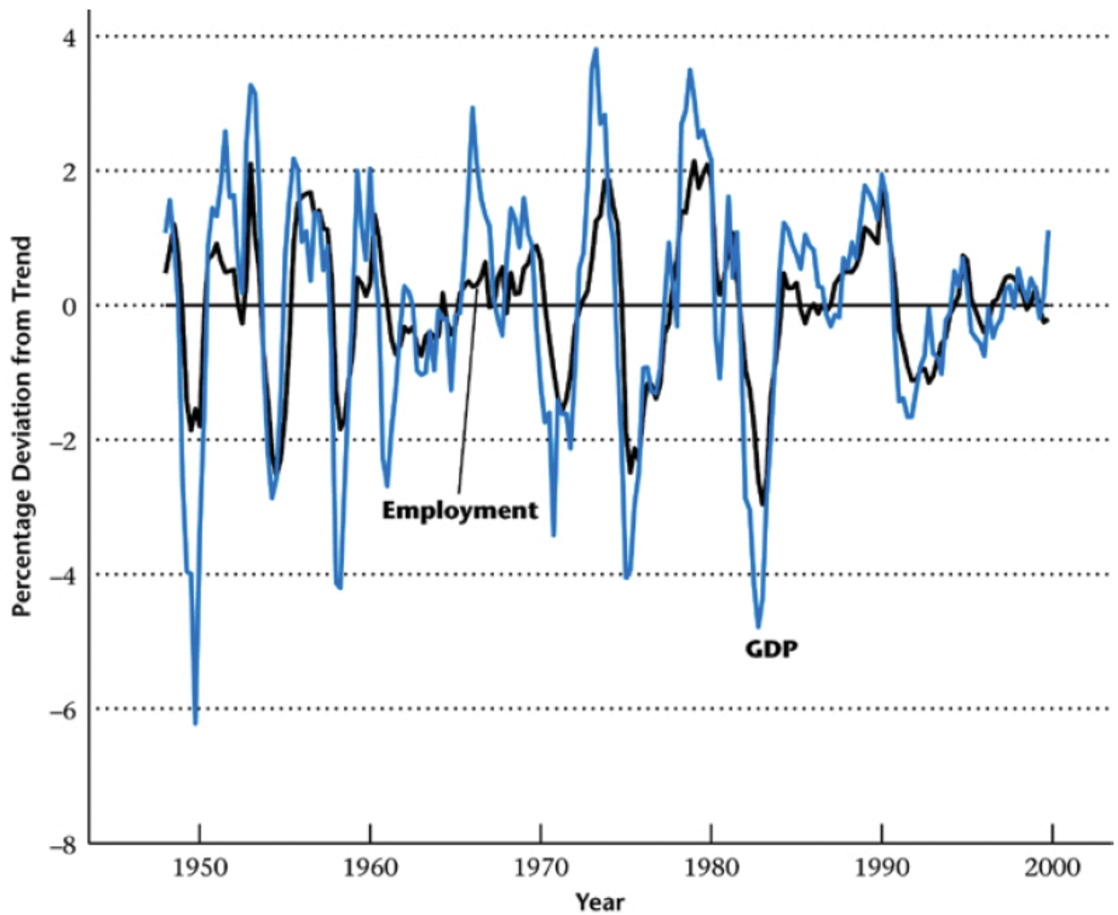
Figure 1

Real GDP and Hours*

H-P Deviations from the Trend



* Output is 1960:1-1994:
Hours is 1964:1-1994:4

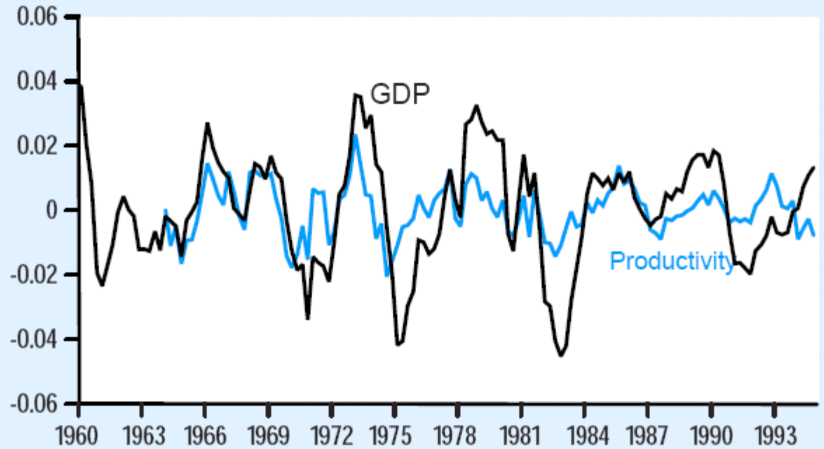


4
2
0
−2
−4
−6
−8
Percentage Deviation from Trend
Employment
GDP
1950
1960
1970
1980
1990
2000
Year

Figure 2

Real GDP and Productivity*

H-P Deviations from the Trend

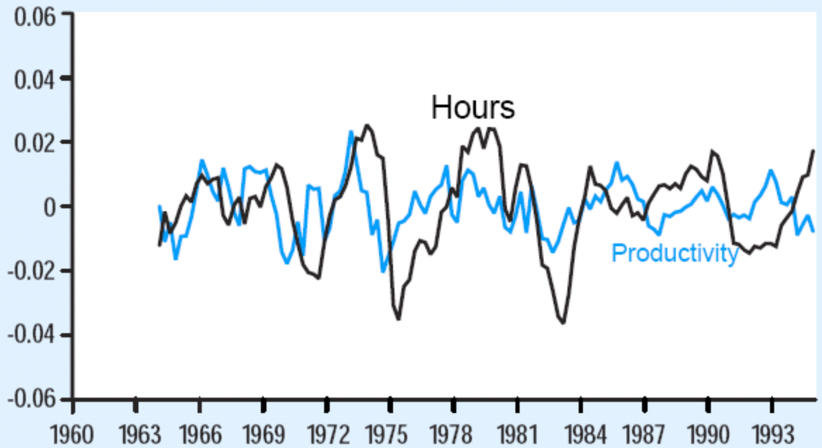


* Output is 1960:1-1994:4
Productivity is 1964:1-1994:4

Figure 3

Hours and Productivity*

H-P Deviations from the Trend



* Hours is 1964:1-1994:4
Productivity is 1964:1-1994:4

Relative fluctuations

- The magnitude of fluctuations in output and aggregate hours of work are nearly equal.
- Employment fluctuates almost as much as output.
- Productivity is slightly procyclical but varies less than output.
- Consumption of non-durables and services is much smoother than output.

- Investment in both producers' and consumers' durables fluctuations more than output.

- Wages vary less than productivity and therefore output.

Co-movement over the Business Cycle

- Consumption, investment, inventories and imports are strongly procyclical, but exports largely acyclical.

- Government expenditures are essentially uncorrelated with output.

- Average hourly compensation is uncorrelated with output.

Table 1

Cyclical Behavior of U.S. Production Inputs

Deviations From Trend of Input Variables
Quarterly, 1954–1989

Variable x	Volatility (% Std. Dev.)	Cross Correlation of Real GNP With										
		x(t–5)	x(t–4)	x(t–3)	x(t–2)	x(t–1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
Real Gross National Product	1.71	–0.03	0.15	0.38	0.63	0.85	1.00	0.85	0.63	0.38	0.15	–0.03
Labor Input												
Hours (Household Survey)	1.47	–0.10	0.05	0.23	0.44	0.69	0.86	0.86	0.75	0.59	0.38	0.18
Employment	1.06	–0.18	–0.04	0.14	0.36	0.61	0.82	0.89	0.82	0.67	0.47	0.25
Hours per Worker	0.54	0.08	0.21	0.35	0.49	0.66	0.71	0.59	0.43	0.29	0.11	–0.02
Hours (Establishment Survey)	1.65	–0.23	–0.07	0.14	0.39	0.66	0.88	0.92	0.81	0.64	0.42	0.21
GNP/Hours (Household Survey)	0.88	0.11	0.21	0.34	0.48	0.50	0.51	0.21	–0.02	–0.25	–0.34	–0.36
GNP/Hours (Establishment Survey)	0.83	0.40	0.46	0.49	0.53	0.43	0.31	–0.07	–0.31	–0.49	–0.52	–0.50
Average Hourly Real Compensation (Business Sector)	0.91	0.30	0.37	0.40	0.42	0.40	0.35	0.26	0.17	0.05	–0.08	–0.20
Capital Input												
Nonresidental Capital Stock*	0.62	–0.58	–0.61	–0.51	–0.48	–0.31	–0.08	0.16	0.39	0.56	0.66	0.70
Structures	0.37	–0.45	–0.51	–0.55	–0.53	–0.44	–0.29	–0.10	0.09	0.25	0.38	0.45
Producers' Durable Equipment	0.99	–0.57	–0.58	–0.53	–0.41	–0.22	0.02	0.26	0.47	0.62	0.70	0.71
Inventory Stock (Nonfarm)	1.65	–0.37	–0.33	–0.23	–0.05	0.19	0.50	0.72	0.83	0.81	0.71	0.53

*Based on quarterly data, 1954:1–1984:2.
Source of basic data: Citicorp's Citibase data bank

Table 2
Cyclical Behavior of U.S. Output and Income Components
Deviations From Trend of Product and Income Variables
Quarterly, 1954–1989

Variable x	Volatility (% Std. Dev.)	Cross Correlation of Real GNP With										
		$x(t-5)$	$x(t-4)$	$x(t-3)$	$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$	$x(t+4)$	$x(t+5)$
Real Gross National Product	1.71	−0.03	0.15	0.38	0.63	0.85	1.00	0.85	0.63	0.38	0.15	−0.03
Consumption Expenditures	1.25	0.25	0.41	0.56	0.71	0.81	0.82	0.66	0.45	0.21	−0.02	−0.21
Nondurables & Services	0.84	0.20	0.38	0.53	0.67	0.77	0.76	0.63	0.46	0.27	0.06	−0.12
Nondurables	1.23	0.29	0.42	0.52	0.62	0.69	0.69	0.57	0.38	0.16	−0.05	−0.22
Services	0.63	0.03	0.25	0.46	0.63	0.73	0.71	0.60	0.49	0.39	0.23	0.07
Durables	4.99	0.25	0.38	0.50	0.64	0.74	0.77	0.60	0.37	0.10	−0.14	−0.32
Investment Expenditures	8.30	0.04	0.19	0.39	0.60	0.79	0.91	0.75	0.50	0.21	−0.05	−0.26
Fixed Investment	5.38	0.09	0.25	0.44	0.64	0.83	0.90	0.81	0.60	0.35	0.08	−0.14
Nonresidential	5.18	−0.26	−0.13	0.05	0.31	0.57	0.80	0.88	0.83	0.68	0.46	0.23
Structures	4.75	−0.40	−0.31	−0.17	0.03	0.29	0.52	0.65	0.69	0.63	0.50	0.34
Equipment	6.21	−0.18	−0.04	0.14	0.39	0.65	0.85	0.90	0.81	0.62	0.38	0.15
Residential	10.89	0.42	0.56	0.66	0.73	0.73	0.62	0.37	0.10	−0.15	−0.34	−0.45
Government Purchases	2.07	0.00	−0.03	−0.03	−0.01	−0.01	0.05	0.09	0.12	0.17	0.27	0.34
Federal	3.68	0.00	−0.05	−0.08	−0.09	−0.09	0.02	0.03	0.06	0.10	0.19	0.24
State & Local	1.19	0.06	0.10	0.17	0.25	0.26	0.25	0.20	0.16	0.19	0.27	0.36
Exports	5.53	−0.50	−0.46	−0.34	−0.14	0.11	0.34	0.48	0.53	0.53	0.53	0.45
Imports	4.92	0.11	0.18	0.30	0.45	0.61	0.71	0.71	0.51	0.28	0.03	−0.19
Real Net National Income												
Labor Income*	1.58	−0.18	−0.02	0.18	0.42	0.68	0.88	0.90	0.80	0.62	0.40	0.19
Capital Income**	2.93	0.10	0.24	0.44	0.63	0.79	0.84	0.60	0.30	0.02	−0.19	−0.29
Proprietors' Income & Misc.†	2.70	0.11	0.24	0.38	0.55	0.62	0.68	0.46	0.29	0.11	0.02	−0.10

Question

- To what extent can business cycle fluctuations be accounted for by a model economy populated by rational (but not perfect foresight) agent responding optimally to real shocks (to, e.g., technology, government purchase, taxes taste, government regulation, terms of trade, energy price, etc.), given their information set, and then re-optimizing when new information become available?

3 The Standard RBC Theory

3.1 Endogenizing Labor Supply

One period example

- The single household has no initial wealth

$$
\begin{aligned}
& \max_{c,h} \log c + \psi \frac{(1-h)^{1-\theta} - 1}{1-\theta} \\
& s.t \\
c \ \leq \ & wh
\end{aligned}
$$

where $h \in [0, 1]$ is the total number of hours the household works.

Normalize total time household has available in a period to 1

- ψ determines how painful it is for the representative household to work.

- Lagrangian

$$L = \log c + \psi \frac{(1-h)^{1-\theta} - 1}{1-\theta} - \lambda (c - wh)$$

- Focs

$$\frac{1}{c} = \lambda \quad (1)$$

$$\psi (1-h)^{-\theta} = \lambda w \quad (2)$$

- $$c = wh = \frac{1}{\lambda}$$

  where first equality comes from the budget constraint and the second from (1). Substitute result in (2) so that

  $$\psi \left(1 - h\right)^{-\theta} = \frac{1}{wh} w = \frac{1}{h}$$

  This means that labor leisure choice is not affected by the level of wage.

WHY?

- (Intratemporal) substitution effect: higher w makes labor more productive, thus effectively leisure more expensive (because its opportunity costs increases). Hence, household substitutes consumption for leisure and work more.

- Income effect: higher w makes economy generate the more output with the same input. Hence, it is optimal to increase both consumption and leisure.

- An increase in wage rate leads to positive income and substitution effect on consumption. For leisure, negative substitution effect and positive income effect.

- Log utility on consumption implies that income and substitution effects for leisure (or labor) cancel each other.

Intertemporal substitution of labor supply

- With log utility for consumption, labor supply does not respond to a permanent increase in wage rate.

- How agents respond to temporarily high wage rate?

- Will show that it is optimal to intertemporally substitute labor supply: work harder when they are more productive. The magnitude depends on the utility function (labor supply elasticity)

A Two-Period Extension

- Life time budget constraint is now

$$c_1 + \frac{c_2}{1+r} = w_1 h_1 + \frac{w_2 h_2}{1+r}$$

- Lifetime utility is

$$\max_{c_1, h_1, c_2, h_2} \log c_1 + \psi \frac{(1-h_1)^{1-\theta} - 1}{1-\theta} + \beta \left(\log c_2 + \psi \frac{(1-h_2)^{1-\theta} - 1}{1-\theta} \right)$$

- Calculating FOCs using Lagrangian method and rearranging terms

$$\begin{aligned} \frac{1}{c_1} &= \beta\left(1+r\right)\frac{1}{c_2} \\ \frac{1-h_2}{1-h_1} &= \left[\beta\left(1+r\right)\frac{w_1}{w_2}\right]^{\frac{1}{\theta}} \end{aligned} \qquad (3)$$

Equation (3) shows that

- The relative labor supply in the two periods responds to relative wage. If w_1 is larger than w_2 (household expects that future productivity will be lower than the current productivity), household supplies more labor today than in the next period. In other words, household has a stronger incentive to substitute consumption for leisure in the first period due to higher w_1.

- The relative labor supply in the two periods responds to the interest rate. A higher interest rate induces household to increase her labor supply today as the return to saving is higher.

- The sensitivity of the responses decreases with θ (or increase with the intertemporal elasticity of substitution, defined as $\frac{1}{\theta}$).

3.2 Introduction of Technology Shocks

- Production function

$$Y_t = e^{z_t} k_t^{\alpha} h_t^{1-\alpha}$$

- Basic idea: make z_t (total factor productivity) vary over time.

- If current TFP is high, output will be high even if inputs stay the same. In addition, household will respond to temporarily higher TFP by working harder, increasing output even further.

- Thus the economy starts to display fluctuations that looks like business cycle driven by fluctuation in z_t, or TFP, and further amplified by endogenous response of labor supply.

Effects of a favorable technology shock on labor supply-through increase in w and r

- Temporary wage increase leads to increase in labor supply due to intertemporal substitution effect on labor supply.
  - the more transitory the shock is, the stronger is the intertemporal substitution effect on labor supply. Hence, labor supply responds more strongly to an increase in w, vise versa.
  - In other words, the more transitory is the shock, the smaller the increase in C, the smaller is the income effect. Hence, labor supply responds more strongly to an increase in w.
- Increase in r makes savings more attractive. Hence labor supply increases in response to a positive technology shock.

3.3 The Baseline Business Cycle Model

- The economy consists of a representative price taking firm and a representative price taking, infinitely-lived household.

- The representative household owns the capital stock k_t and maximizes her expected lifetime utility.

Timing within time t is

1. The beginning-of-period stock of capital is k_t, and the shock is realized.

2. Spot market opens. Firm demands capital services and labor. The household supplies these factors. Market clears for both factors at price r_t and w_t

3. The representative household allocates her total income optimally between consumption and investment.

The Representative Household's Problem

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \psi \log\left(1 - h_t\right)\right]$$

subject to

$$\begin{aligned} k_{t+1} + c_t &= w_t h_t + \left(1 + r_t\right) k_t, \forall t \\ c_t &\geq 0, h_t \in [0,1], k_0 \text{ given} \\ z_t &= \rho z_{t-1} + \varepsilon_t \end{aligned}$$

- Exploit 1st welfare theorem: this economy is Pareto optimal, implying that we can reformulate the problem as a social planner problem

- The resource constraint of the planner is given by

$$k_{t+1} + c_t = (1 - \delta)\, k_t + e^{z_t} k_t^{\alpha} h_t^{1-\alpha}$$

- Note that resource constraint has to hold for every possible realization of the TFP shock.

Social Planner's Problem

$$\max_{\{c_t, h_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \psi \log\left(1 - h_t\right)\right]$$

subject to

$$\begin{aligned}
k_{t+1} + c_t &= \left(1 - \delta\right) k_t + e^{z_t} k_t^{\alpha} h_t^{1-\alpha} && (4) \\
c_t &\geq 0, h_t \in [0, 1] \text{ and } k_0 \text{ given} && (5) \\
z_t &= \rho z_{t-1} + \varepsilon_t && (6)
\end{aligned}$$

Optimality Conditions

- Intratemporal Optimality Conditions

$$\frac{\psi}{1 - h_t} = \frac{(1-\alpha)\, e^{z_t} k_t^{\alpha} h_t^{-\alpha}}{c_t}$$

  state that social planner equates costs and benefits of an extra hour of work. utility cost of a marginal increase in labor is $\frac{\psi}{1-h_t}$.Benefit is to increase production and thus consumption by marginal product of labor, and thus utility from consumption by $\frac{(1-\alpha_t)e^{z_t}k_t^{\alpha}h_t^{-\alpha}}{c_t}$.

- Intertemporal Optimality Condition

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(1 + \alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} - \delta \right) \right]$$

Back to Competitive Equilibrium

- In a decentralized competitive equilibrium, factor prices are equal to value of their respective marginal product, thus

$$\begin{aligned} r_t &= \alpha e^{z_t} k_t^{\alpha-1} h_t^{1-\alpha} - \delta \\ w_t &= (1-\alpha) e^{z_t} k_t^{\alpha} h_t^{-\alpha} \end{aligned}$$

Social Planner's Problem in Recursive Form

- State variables: k_t (endogenous), z_t (exogenous)

- Control variables: c_t, h_t, k_{t+1}

- Bellman Equation

$$V\left(z_t, k_t\right) = \max_{c_t, h_t} \left\{ \begin{array}{c} \log\left(c_t\right) + \psi \log\left(1 - h_t\right) \\ +\beta E_t V\left(z_{t+1}, \left(1-\delta\right) k_t + e^{z_t} k_t^{\alpha} h_t^{1-\alpha} - c_t\right) \end{array} \right\}$$

Focs

$$\frac{1}{c_t} = \beta E_t \frac{\partial V(z_{t+1}, k_{t+1})}{\partial k_{t+1}} \tag{7}$$

$$\frac{\psi}{1 - h_t} = \beta E_t \frac{\partial V(z_{t+1}, k_{t+1})}{\partial k_{t+1}} (1 - \alpha) \frac{y_t}{h_t} \tag{8}$$

- Note that (7) and (8) implies the intratemporal optimality condition.

- or more generally

$$-u_h(c_t, h_t) = (1 - \alpha) \frac{y_t}{h_t} u_c(c_t, h_t)$$

The envelop condition

$$\frac{\partial V}{\partial k_t}(z_t, k_t) = \beta E_t \left[\frac{\partial V(z_{t+1}, k_{t+1})}{\partial k_{t+1}} \right] R_t \tag{9}$$

where $R_t = 1 - \delta + \alpha \frac{y_t}{k_t}$

- Plug (7) and (8) into (9)

$$\begin{aligned}
\frac{\partial V}{\partial k_t}(z_t, k_t) &= \frac{R_t}{c_t} \\
\frac{\partial V}{\partial k_t}(z_t, k_t) &= \frac{\psi}{1 - h_t} \frac{h_t}{(1 - \alpha) y_t} R_t
\end{aligned}$$

Euler Equations

- Taking one period ahead and conditional expectations

$$1 = \beta E_t \left[\frac{c_t}{c_{t+1}} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right]$$
$$1 = \beta E_t \left[\frac{1-h_t}{1-h_{t+1}} \frac{h_{t+1}}{y_{t+1}} \frac{y_t}{h_t} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right]$$

- The first equation governs the intertemporal trade-off of consumption.

- The second governs the intertemporal trade-off of hours works (leisure).