RBC Model with Indivisible Labor

Advanced Macroeconomic Theory

Last Class

- What are business cycles?
- Using HP-filter to decompose data into trend and cyclical components
- Business cycle facts
- Standard RBC theory

Road map for this lecture

- Calibration

- Quantitative performance of the Standard RBC theory

- RBC Model with Indivisible Labor

1 Calibration

- Parameters of the model to be calibrated
  - technology parameters: $(\alpha, \delta, \rho, \sigma_\varepsilon)$
  - Preference parameters: (ψ, β)
- Parameters that have a time dimension: δ, β

- Process of choosing these parameters is often called calibration.

- Idea: we want to make the model's predictions to match certain observations from the data.

- Since we are interested in business cycle properties of the model and do not want to cheat by choosing parameter values that help the model deliver good business cycle implications, we choose parameters to match long run observations from the data

- Calibration is country specific. We do it for the U.S.

- Need to specify period length, because some parameter values have time dimension.

- Since most business cycle research is done with quarterly data, we use a quarter as the length of a period in the model.

- For those parameters that have a time dimension, we compute the annual values and convert them to the quarterly values.

- Alternatively, we can directly compute the quarterly values, with the flow variables, say, I, Y, C, adjusted to their quarterly values when calibrating.

Capital and labor share

- In our model, production function parameter α equals the share of capital income in total output.

- Thus choose a value for α that corresponds to the long run capital share in the data.

- Choose $\alpha = .40$, reflecting the long run labor income share of roughly 0.60. The range of estimates used in the literature is $[0.25, 0.40]$.

Depreciation rate

- Suppose the economy is at the steady state.

- Divide both sides of the law of motion for k by k

$$1 = (1 - \delta) + \frac{I}{K}$$

- The long run average ratio of $\frac{I}{K}$ is 0.076. This yields an annual $\delta = 0.076$, or a quarterly value of $\delta = 0.02$

Preference Parameters

- for β, use the steady state version of Euler equation

$$1 = \beta \left(\alpha \frac{y}{k} + 1 - \delta \right)$$

- capital output ratio is estimated to be 3.32. This yield an annual value for β of 0.96, or a quarterly value of 0.99

- ψ is calibrated to match the fact that on average people work a third of their non-sleeping time to market activity

- The social planner's intratemporal optimality condition implies

$$\frac{1-h}{h} = \frac{\psi}{1-\alpha}\frac{c}{y}$$

- The $\frac{c}{y}$ is estimated to be 0.75, and $h = 0.31$ (people work a third of their non-sleeping time for market activity). $\psi = 1.78$.

Technology Shocks

- $z_{t+1} = \rho z_t + \varepsilon_{t+1}$. Need to assign value to ρ and σ_ε^2.

  $z_{t+1} - z_t = \log Y_{t+1} - \log Y_t - \alpha (\log K_{t+1} - \log K_t) - (1-\alpha)(\log H_{t+1} - \log H$

- This gives us a series for z_t (given $z_0 = 0$) . We can run OLS regression to estimate ρ and σ_ε^2

- $\rho = 0.95$ and $\sigma_\varepsilon = 0.007$

- Solve the decision rules numerically using value function iteration or other methods.

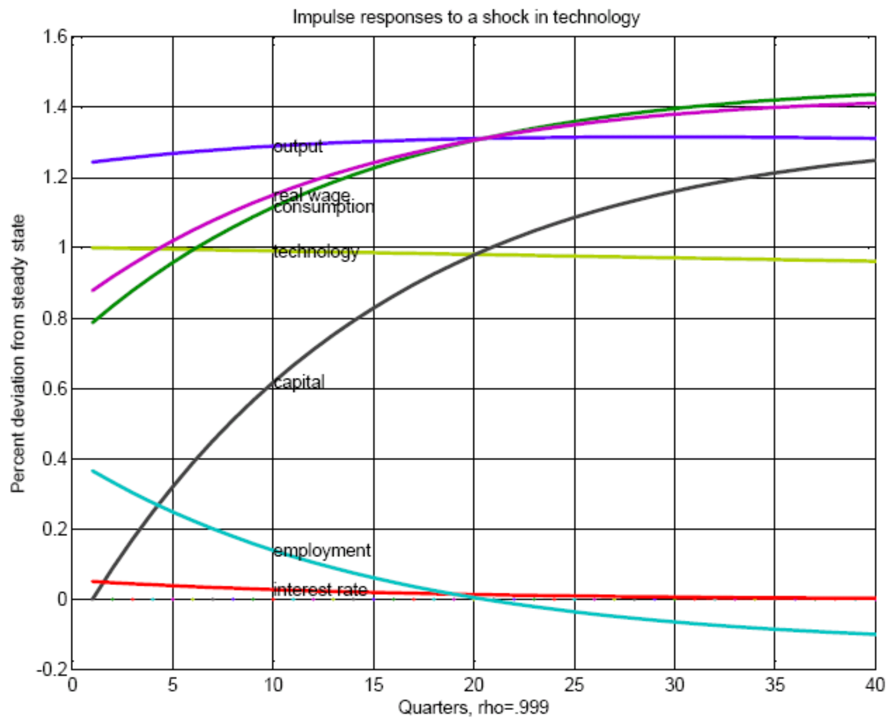
2 Quantitative properties of the Model

Impulse response function

- Assume that before period 0, the economy is at steady state.
- At period 0, there is a positive technology shock, which increases technology by 1%, that is $z_0 = 0.01$.
- After this shock, the technology is by assumption not hit by further shocks, and thus the technology level follows the process

$$z_t = \rho z_{t-1}$$

- Impulse response functions trace out how endogenous variables respond to the shock over time.



Source: Blanchard (2002)

Main Observations

- Labor supply responds positively to the increase in productivity. As a result, on impact output increases by more than the technology shock.
- Consumption is hump-shaped, because it is optimal to devote a lot of extra output to investment initially due to an increase in interest rate.
- Eventually technology goes back to the steady state, so do output, consumption, labor and capital.
- Persistent effect of output in this model is due to

– persistent technology shock

– increase in capital stock.

Comparing Business Cycle Statistics of the Model with the Data

- Basic Idea
  - Start with initial condition for capital stock k_0 (normally equal to the steady state value) and for $z_0 = 0$.
  - Draw a sequence of technology shocks $\{\varepsilon_t\}_{t=0}^{T}$, and construct $\{z_t\}_{t=0}^{T}$ from the equation

$$z_t = \rho z_{t-1} + \varepsilon_t$$

  - Simulate a large number of realizations of the model economy 100 times, each simulation being 150 period long (which is the length of the periods in data)

– decompose both the artificial and real data into trend and cyclical components with H-P filter or other methods.

– With cyclical components of artificial data from the model can compute exactly the same statistics as we did from the real data.

x	$\sigma_x\,(m)$	$\sigma_x\,(d)$	$\sigma_x/\sigma_y\,(m)$	$\sigma_x/\sigma_y\,(d)$	$\rho_{x,y}\,(m)$	$\rho_{x,y}\,(d)$
Output	1.351	1.72	1	1	1	1
Consumption	0.329	1.27	0.244	0.738	0.84	0.83
Investment	5.954	8.24	4.407	4.791	0.99	0.91
Hours	0.769	1.65	0.569	0.930	0.99	0.86

- Output fluctuates less in the model that in the U.S. economy (the volatility predicted by the model is 70-75% of that in the data)

- Labor input fluctuates only about half as much as in the U.S. economy.

- Investment fluctuates much more than output and consumption much less than output. Relative to GDP, consumption fluctuate significantly more in the data than in the model.

- The correlation of all variables with output is very high, and higher than that in the data.

- The model predicts a very strong correlation between the labor input and labor productivity.

Underlying reasons for model weakness

- The technology shock by assumption is very persistent (to ensure the persistence of output). Thus wage adjusts very smoothly, implying small intertemporal substitution in labor supply.

- The movement in labor supply are instead mainly associated with changes in interest rate, Hence the volatility of hours is small compared to the volatility of output.

- In this frictionless economy, it is too easy to smooth consumption over time with saving alone.

- The high correlation of all variables with output is due to the fact that there is only one shock in this economy.

Problems and Potential Solutions

- Main failure: too little hours fluctuation.

- Potential solutions:

  - abandon the logarithm specification and allow the intertemporal elasticity of labor supply to take value larger than one.

  - recognize explicitly that most (about two thirds) of the fluctuation in labor supply comes from changes along the extensive margin (movement into and out of employment) rather than along the *intensive margin* (changes in hours when employed).

3 RBC Model with Indivisible Labor

We need a volatile labor input fluctuation over the business cycle

- period utility in general form

$$\log(c_t) + \psi \frac{(1 - h_t)^{1-\theta} - 1}{1 - \theta}$$

- Frisch elasticity of leisure (labor supply): $\frac{1}{\theta}$.

  - the elasticity of the leisure (labor supply) with respect to wage leaving constant the marginal utility of consumption.

- More volatile fluctuation of labor input requires a higher Frisch elasticity.

- Maximum Frisch elasticity obtained when $\theta = 0$. The period utility

$$\log(c_t) - \psi h_t$$

  - Marginal disutility is labor is linear, implying that labor supply reacts strongly to changes in wages

- How can we justify that marginal disutility of labor is constant at ψ?

Indivisible labor

- There is a continuum of ex-ante identical agents.

- Suppose the period utility function is given by

$$u(c_t, h_t) = \log(c_t) + v(1 - h_t)$$

where the function v satisfies $v' > 0$, $v^{''} < 0$.

- Assume that household can either work full time, $h_t = \widehat{h}$, $0 < \widehat{h} < 1$,or not at all $h_t = 0$. That is, they either have a job requiring fixed number of working hours, or they don't.

- Rationale for this assumption: in the U.S. economy, about two thirds of variations in hours worked comes from individuals moving into and out of unemployment, with only one third from variations in hours when employed.

- However, European data displays greater variance in hours worked per worker than in the number of workers.

Labor lottery

- Now, assume each individual can pick a probability π_t that he is employed and works $\widehat{h}$ hours in period $t, \pi_t \in [0,1]$. (This will make individuals happier than in the case $\pi_t = \{0,1\}$.)
  - Since all agents are *ex-ante* the same, they will choose the same probability π_t.

- Hence, π_t is also the fraction of agents that are employed each period.

- A lottery determines who will be actually be unemployed at each period.

- Individuals can insure each other against the contingency of unemployment.

The social planning problem

- Alternatively, we can let the social planner choose π_t, the fraction of population to work at each period.

- Assume that all people get picked to work $\widehat{h}$ hours with the same probability, so π_t is also the probability that a particular agent get picked.

- Denote $H_t = \pi_t \widehat{h}$ as the hours worked per capita.

- Assume that the social planner provides full insurance against being unemployed.

- As part of the dynamic social planning problem, the social planner solves

$$\max_{c_{1t},c_{0t}} \pi_t \left(\log c_{1t} + v\left(1-\widehat{h}\right)\right) + (1-\pi_t)\left(\log c_{0t} + v(1)\right),$$

  subject to

$$\pi_t c_{1t} + (1-\pi_t)\, c_{0t} = c_t$$

  where c_t is total per capita consumption, which is given at this stage, c_{1t} (c_{0t}) is consumption of an employed (unemployed) agent.

- The solution to the above problem is $c_{1t} = c_{0t} = c_t$.

- The current period expected utility becomes

$$\begin{aligned} Eu(c_t, h_t) &= \pi_t \left(\log c_t + v\left(1 - \widehat{h}\right)\right) + (1 - \pi_t)(\log c_t + v(1)) \\ &= \log c_t - \pi_t \left(v(1) - v\left(1 - \widehat{h}\right)\right) + v(1) \end{aligned}$$

- Ignoring constants added to the utility function, we can rewrite the effective utility function as

$$Eu(c_t, h_t) = \log c_t - \psi H_t$$

- where $\psi = \left(v(1) - v\left(1 - \widehat{h}\right)\right) / \widehat{h} > 0$.

- Therefore, the decision variables for the social planner are the same as for a divisible labor model

$$\max_{\{c_t, H_t, k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi H_t\right)$$

subject to

$$\begin{aligned} k_{t+1} + c_t &= (1-\delta) k_t + e^{z_t} k_t^{\alpha} H_t^{1-\alpha} && (1) \\ c_t &\geq 0, H_t \in [0,1] \text{ and } k_0 \text{ given} && (2) \\ z_t &= \rho z_{t-1} + \varepsilon_t && (3) \end{aligned}$$

Key assumptions and implications

- Key Assumptions
  - Full unemployment insurance.
  - All agents are ex-ante homogeneous.
- Fluctuations in labor input comes from fluctuations in employment rather than fluctuations in hours per employed worker.
- At the aggregate level, the Frisch elasticity of labor supply is infinity, though for a continuously employed worker, hours worked are constant (implying a value of 0 for Frisch elasticity of labor supply at the micro level).

A simple example

- Agents live for two periods, don't discount the future and only value consumption in the second period.
  - Abstract from capital accumulation, but let households store output between the first and the second period.
- Let A_1 and A_2 denote labor productivity in the first and second period (which is also the wage rate in this case).
- Social planner's problem

$$\begin{aligned} & \max \log c_2 - \psi h_1 - \psi h_2 \\ s.t.\ c_2 &= A_1 h_1 + A_2 h_2 \end{aligned}$$

- Optimal Choice: if $A_1 > A_2$, then the agent should worked only in period 1. *Vise Versa.*

- Extra unit of work brings about an extra disutility of work ψ, no matter when it is done. Thus should always work that extra unit in the period when labor is more productive.

- In this case, $h_2 = 0$, $h_1 = \frac{1}{\psi}$ and $c_2 = \frac{A_1}{\psi}$.

- For linear disutility of labor, effects of changes in productivity on labor supply are very strong.

- Suppose the utility is

$$\log c_2 + \psi \log(1 - h_1) + \psi \log(1 - h_2)$$

- Optimality condition

$$\frac{A_2}{A_1} = \frac{1 - h_1}{1 - h_2}$$

- Labor supply does not respond as drastically to difference in A_1 and A_2. As long as $\frac{A_2}{A_1}$ not too big, work in both period. Small changes in $\frac{A_2}{A_1}$ do not lead to drastic labor supply responses.

- Data: labor input varies substantially over cycle, real wages only moderately. Linear specification more successful.

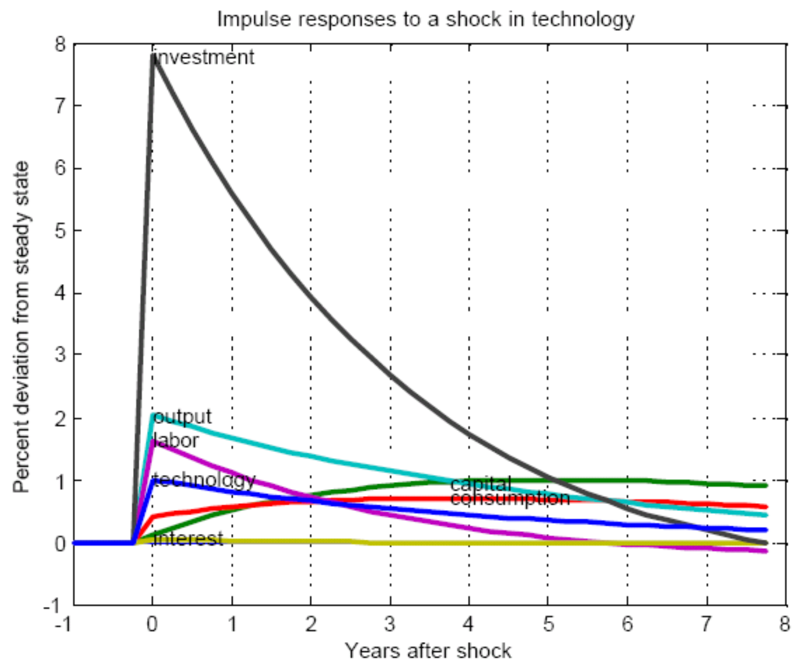
Calibration of the RBC model with indivisible labor

- pick ψ so that the model economy reproduce amount of work equal to long run average in the data (about one third of their non-sleeping time).
  - Compute the stead state hours worked.
  - Back out ψ such that $H = \frac{1}{3}$.

- The calibration of other parameters are the same as before.

3.1 Quantitative Properties of the model

Impulse response functions



Source: *Quantitative Macroeconomics*, By Dirk Krueger

Main observations

- Labor supply responds more than the standard RBC model.
- The response of output is amplified by the larger response of labor supply.

Compare business cycle statistics of the model with the data

Table 3

Cyclical Properties of U.S. and Model-Generated Time Series

Type of Data or Model	% S.D. of Output σ_y	Variable vs. Output: Consumption σ_c/σ_y	Investment σ_i/σ_y	Hours σ_h/σ_y	Productivity σ_w/σ_y	Hours vs. Productivity: σ_h/σ_w	cor(h,w)
U.S. Time Series*							
Output	1.92	.45	2.78	—	—	—	—
Hours Worked:							
1. Household Survey (All Industries)	—	—	—	.78	.57	1.37	.07
2. Establishment Survey (Nonag. Industries)	—	—	—	.96	.45	2.15	–.14
Models**							
Standard	1.30	.31	3.15	.49	.53	.94	.93
Nonseparable Leisure	1.51	.29	3.23	.65	.40	1.63	.80
Indivisible Labor	1.73	.29	3.25	.76	.29	2.63	.76
Government Spending	1.24	.54	3.08	.55	.61	.90	.49
Home Production	1.71	.51	2.73	.75	.39	1.92	.49

*U.S. data here are the same as those in Table 2; they are for the longer time period: 1947:1–1991:3.

**The standard deviations and correlations computed from the models' artificial data are the sample means of statistics computed for each of 100 simulations. Each simulation has 179 periods, the number of quarters in the U.S. data.

Source: Citicorp's Citibase data bank

Source: Hansen and Wright (1992)

- Compared with standard RBC model, models with indivisible labor supply generates hours volatility much close to the data.

- As a result, the volatility of output in this model is closer to the data.

- Also, model with indivisible labor supply somehow lower the correlation between labor input and labor productivity, but still much higher than the data.

- The volatility of consumption relative to output in this model is roughly the same as that in the standard model.

Other Extensions of RBC Models to deal with labor market shortcomings

- Home Production
  - Motivation: empirically, women's elasticity of labor supply along the extensive margin is much higher than men.
  - By introducing home production, increase the intertemporal elasticity of labor supply.

- Government spending shock.
  - Labor supply responds to government spending shock in addition to wages.

  - Somehow lower the correlation between labor supply and labor productivity.

- Labor market search frictions

- Learning by working