# Great Depressions from a Neoclassical Perspective

Advanced Macroeconomic Theory

## Review of Last Class

- Model with indivisible labor, either working for fixed hours or not.
  - allow social planner to choose the fraction of agents to work each period.
  - social planner also provide full insurance against unemployment risk.
  - we shows that the decision variables for the social planner are the same as for a divisible labor model, though the disutility for labor is linear.
  - Frisch elasticity of labor supply is infinite in this model, while in the standard model it is one.

- The model generated fluctuation of labor input very close to the data, through fluctuations in the fraction of people employed rather than fluctuations in hours per employed worker.

- When agents are ex-ante heterogeneous, and there is no full insurance for unemployment risks, the aggregate elasticity of labor supply depend on the distribution of reservation wage.
  - Aggregate elasticity of labor supply tends to be large where there is a large density in reservation wage distribution.

## Road map of this Class

- Great Depression of the Twentieth Century: An Overview

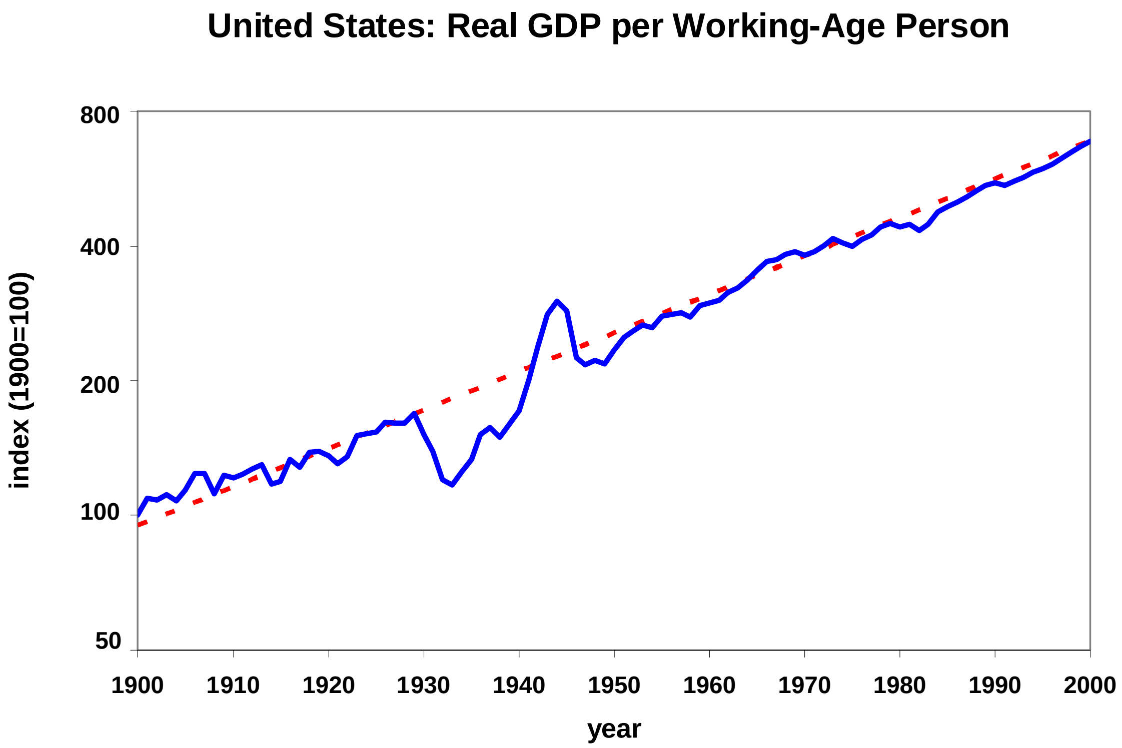
- Great Depression Methodology

  - Growth Accounting

  - Dynamic General Equilibrium Model

  - Diagnose by Measuring First-order Condition Deviations

# 1 Great Depression in the 20th Century


United States: Real GDP per Working-Age Person
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200
100
50
index (1900=100)
1900
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year

# Digression: Economic Growth

- Model so far does not display long-run growth, since the economy converges to is steady state.

- Data do.

- Partly due to population growth, but even GDP per capita (or per working age person) grows at a positive rate.

- Assume working-age population (and labor force) grows at a constant gross growth rate $\eta$.

$$N_t = \eta^t N_0 = \eta^t$$

  where $N_0 = 1$ is the size of labor force at period 0.

- Labor augmenting technological process. Assume that production function is given by

$$Y_t = z_t K_t^{\alpha} \left(\gamma^t H_t\right)^{1-\alpha}$$

  where $\gamma$ is the long-run growth rate of technology, $H_t = h_t N_t$ is aggregate hours worked, $y_t = Y_t / N_t$ is output per working age person. $z_t$ is country-specific productivity parameter that varies over time.

# Balanced Growth Path

- Balanced growth path is an equilibrium or social planner allocation where all per capita variables grow at a constant rate, with the exception of market hours per working age person, $h$, which is constant.

- Easy to show that the constant growth rate has to be $\gamma$.

- Define trend level of output and output per working age population as

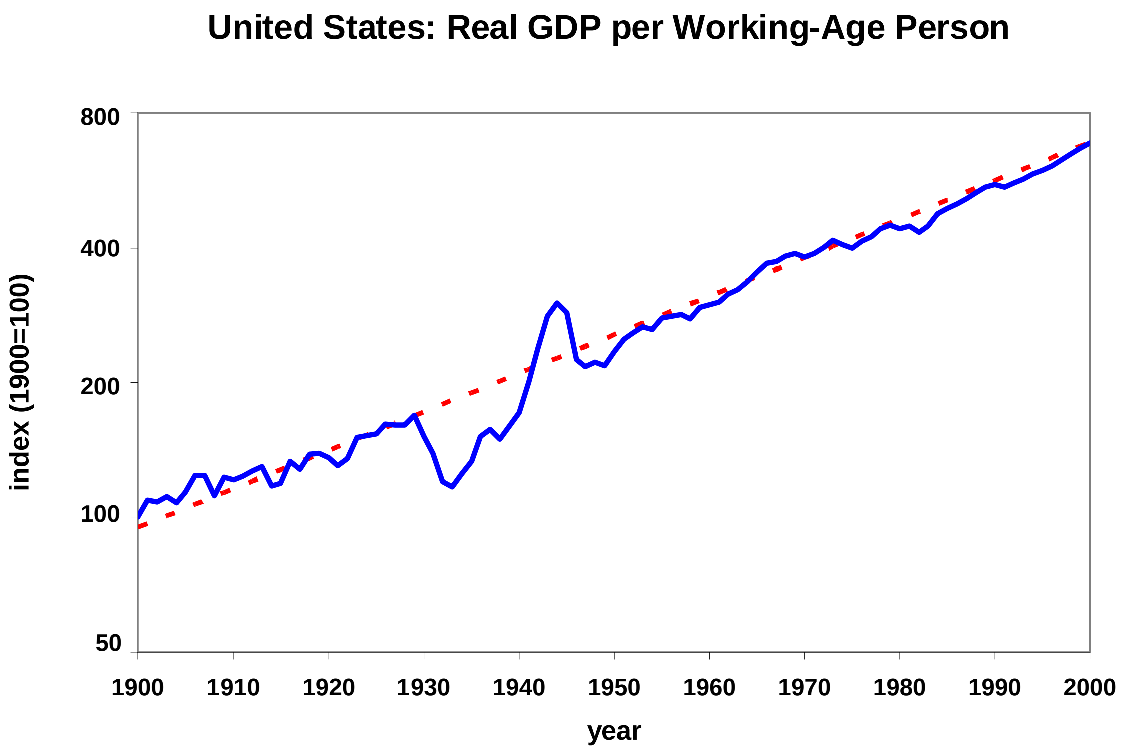
$$\begin{aligned} \widehat{Y}_t^i &= \gamma^t N_t \widehat{Y}_0^i \\ \widehat{y}_t^i &= \gamma^t \widehat{y}_0^i \end{aligned}$$

# Definitions of Great Depressions

- A large negative deviation from trend (or balanced) growth.

- Twentieth century U.S. macro data are very close to a balanced growth path, with the exception of Great Depression and the subsequent World War II built-up.

- Trend growth rate is set to be two percent per year ($\gamma = 1.02$), the long run growth rate of output per working-age person in the United States during the twentieth century.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# Conditions for a negative deviation from trend to be a great depression

- It must be a sufficiently large deviation.
  - A great depression is a deviation of at least 20 percent below trend level.

- The deviation must occur rapidly.
  - Detrended output per working age person must fall at least 15 percent within the first decade of depression.

- A time period $D = [t_0, t_1]$ is a great depression if
  - there is some year $t$ in D such that $\frac{y_t^i}{\gamma^{t-t_0}\widehat{y}_{t_0}^i} - 1 \leq -0.20$.
  - there is some $t_0 \leq t \leq t_0 + 10$ such that $\frac{y_t^i}{\gamma^{t-t_0}\widehat{y}_{t_0}^i} - 1 \leq -0.15$
- We do not require that an economy return to the original trend path at the end of a depression.
  - We would however expect the productivity factor and eventually the economy itself to grow at the trend rate.
- For the starting year of a depression $t_0$, we identify the trend level $\widehat{y}_{t_0}^i$ with the observed level $y_{t_0}^i$.

# An overview of great depressions in the twentieth century

- 1930s: United States, United Kingdom, Canada, France, Germany

- Contemporary: Argentina (1970s and 1980s), Chile and Mexico (1980s), Brazil (1980s and 1990s), New Zealand and Switzerland (1970s, 1980s, and 1990s), Argentina (1998-2002)

- Not-quite-great Depressions: Italy (1930s), Finland (1990s), Japan (1990s)

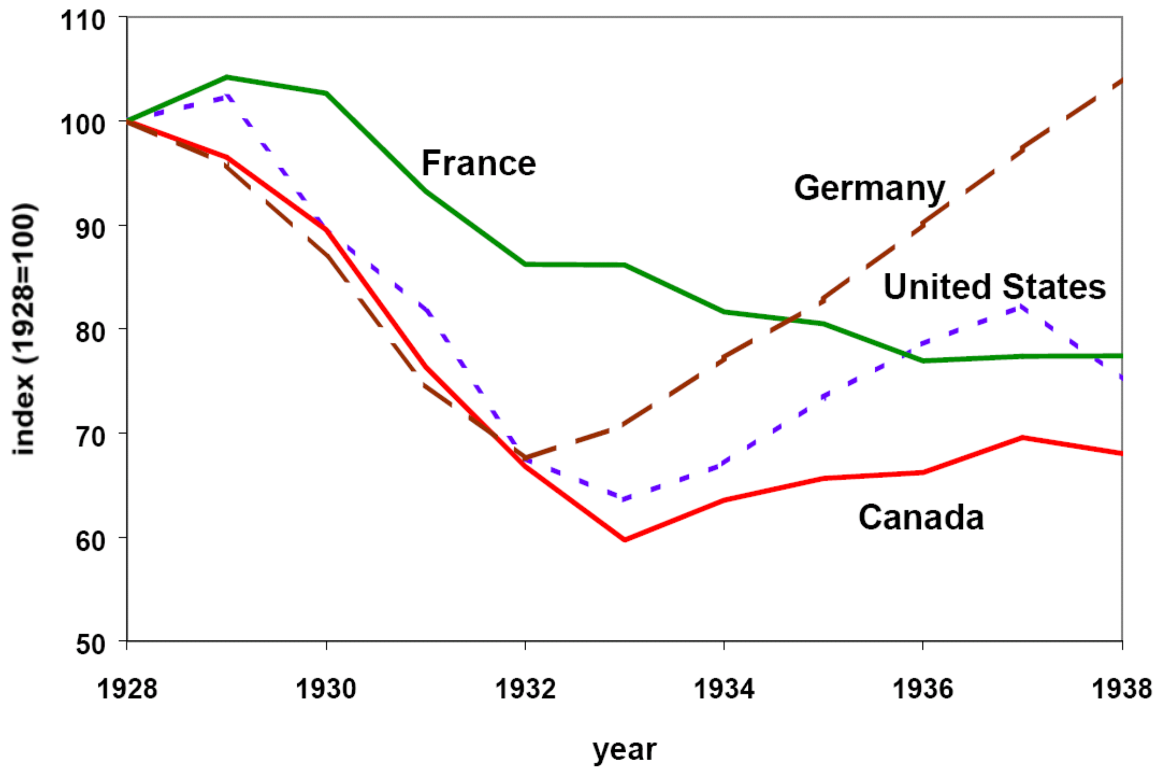



Figure 1: Detrended output per person during the Great Depression

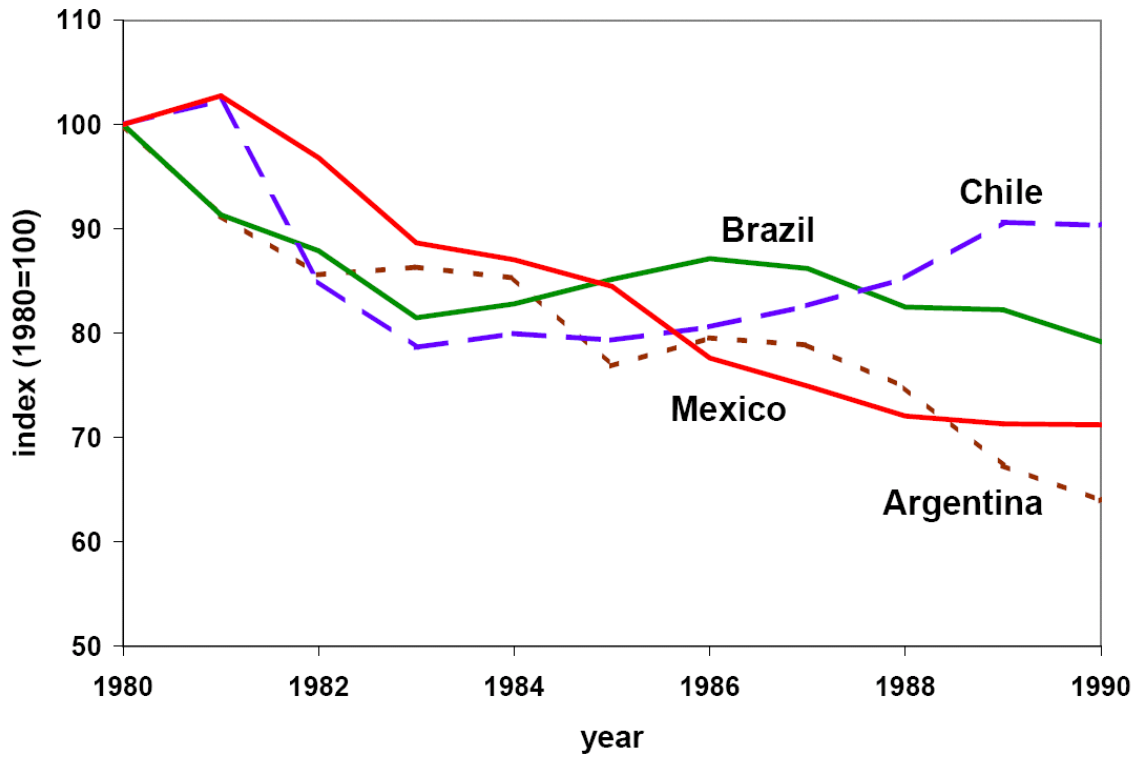



Figure 2: Detrended output per working age person during the 1980s in latin America

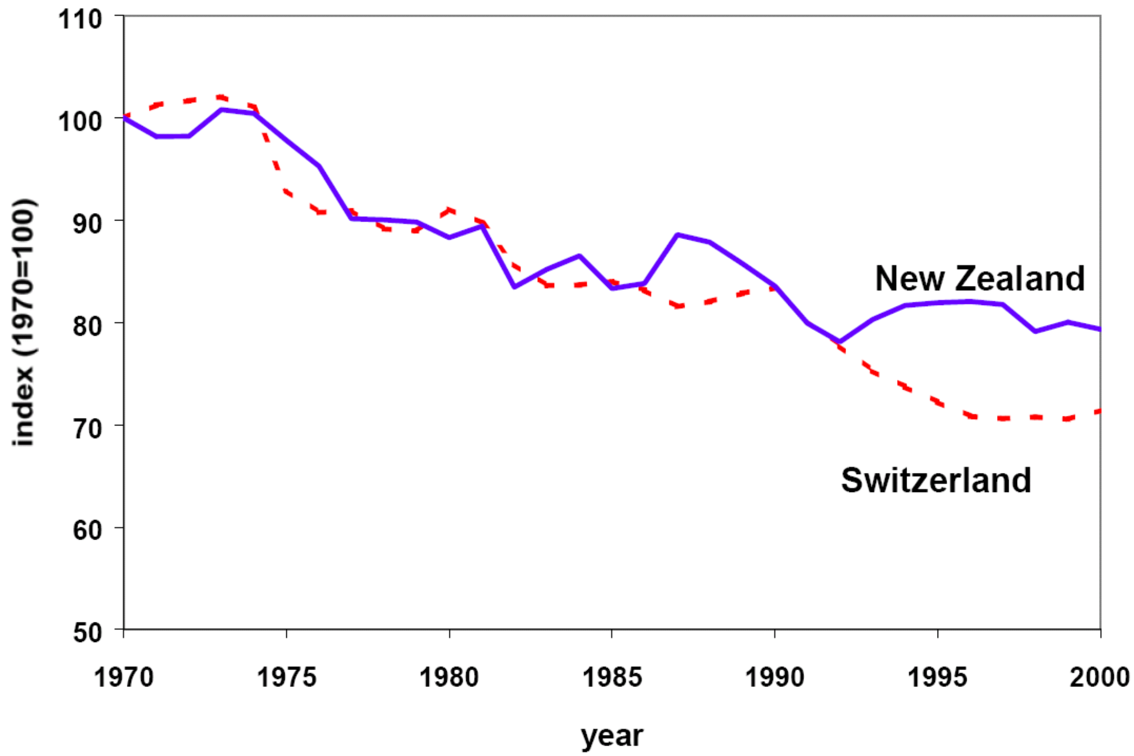



Figure 3: Detrended Output per Working-Age Person in New Zealand and Switzerland (1970-2000)

# 2 Great Depression Methodologies

## 2.1 Growth Accounting

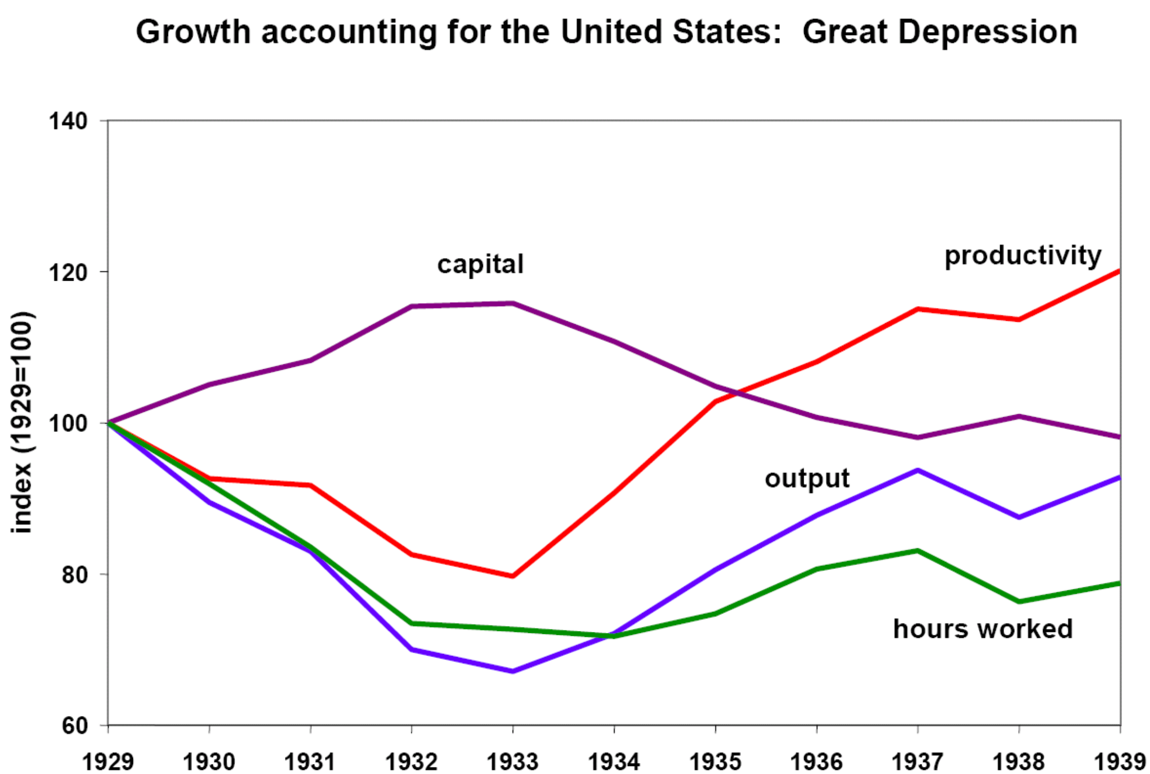
- rewrite the production function

$$\log y_t = t \log \gamma + \frac{1}{1-\alpha} \log z_t + \frac{\alpha}{1-\alpha} \log k_t / y_t + \log h_t$$

  where lower case variables denote per working-age person values of a variable.

- Along the balanced growth path, output per working age person grows at the trend growth rate and each of the remaining three factors are constant.

- Each of the last three factors allows us to examine different set of shocks and changes in policies while studying output.
  - Constraints imposed upon the way businesses operate, such as a restriction on the adoption of a more efficient production technology, will reduce the productivity factor.
  - A change in the tax system that makes consumption more expensive in terms of leisure will reduce the balanced growth value of the labor factor.
  - A change in the tax system that taxes capital income at a higher level will reduce the balanced-growth value of the capital factor.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# Features of U.S. Great Depression

- Output fell more than 38% between 1929 and 1933.
- By 1939, output remained 11 percent below its 1929 detrended level.
- Total factor productivity declines sharply in 1932 and 1933, falling about 12 percent and 14 percent, respectively, below their 1929 detrended level.
- After 1933, TFP rose quickly relative to trend and returned to trend by 1936.

- Total hours plummeted more than 30 percent between 1929 and 1933, and remained 22 percent below their detrended 1929 level at the end of the decade.

## 2.2 Dynamic General Equilibrium Model

- Can neoclassical theory account for the Great Depression in the United States?
  - both the downturn in output between 1929 and 1933 and the recovery between 1934 and 1939.
- We introduce trend growth in technology and population in our model.
- We take the path of productivity factor as exogenous.
- Comparing results of the model with the data, we can identify features of the U.S. Great Depression that need further analysis.

# Social Planner's Problem

$$\max_{\{c_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t N_t \left[\log c_t + \psi \log\left(1 - h_t\right)\right]$$

subject to

$$\begin{aligned} K_{t+1} + C_t &= \left(1 - \delta\right) K_t + z_t K_t^{\alpha} \left(\gamma^t H_t\right)^{1-\alpha} & (1) \\ c_t &\geq 0, h_t \in [0,1] \text{ and } K_0 \text{ given} & (2) \\ z_t &= \left(1 - \rho\right) + \rho z_{t-1} + \varepsilon_t, \varepsilon_t \tilde{} N\left(0, \sigma^2\right) & (3) \end{aligned}$$

where $c_t = \frac{C_t}{N_t}, h_t = \frac{H_t}{N_t}, k_{t+1} = \frac{K_{t+1}}{N_{t+1}}$, and the mean value of $z$ is 1.

- Intratemporal optimality condition

$$\frac{\psi}{1-h_t} = \frac{z_t\left(1-\alpha\right)\left(K_t/H_t\right)^{\alpha}\gamma^{t(1-\alpha)}}{c_t}$$

- Intertemporal optimality condition

$$\frac{1}{c_t} = \beta E_t\left[\frac{1}{c_{t+1}}\left(1-\delta+z_{t+1}\alpha K_{t+1}^{\alpha-1}\left(\gamma^{t+1}H_{t+1}\right)^{1-\alpha}\right)\right]$$

# Detrended variables

$$
\begin{aligned}
\widetilde{c}_t &= \frac{C_t}{\gamma^t N_t} = \frac{c_t}{\gamma^t} \\
\widetilde{y}_t &= \frac{Y_t}{\gamma^t N_t} = \frac{y_t}{\gamma^t} \\
\widetilde{k}_t &= \frac{K_t}{\gamma^t N_t} = \frac{k_t}{\gamma^t}
\end{aligned}
$$

# Rescaling in detrended variables

- Hard to solve for the decision rules numerically in a growing economy.
- Want to rewrite the problem in terms of variables that not constantly growing over time, that is in terms of ˜ variables.
- Note that $K_0 = k_0 = \widetilde{k}_0$, since $\gamma^0 = N_0 = 1$.
- Need to rescale resource constraint and the utility function.

# Rescaling of the utility function

- With the above utility function we have

$$\log c_t + \psi \log(1 - h_t) = \log \tilde{c}_t + \log \gamma^t + \psi \log(1 - h_t)$$

- Can rewrite the lifetime utility of the representative family as

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t N_t \left[\log c_t + \psi \log(1 - h_t)\right] \\ = & \sum_{t=0}^{\infty} \beta^t N_t \left[\log \tilde{c}_t + \psi \log(1 - h_t)\right] + \sum_{t=0}^{\infty} \beta^t N^t \log \gamma^t \end{aligned}$$

- can omit the constant term in utility.

# The Social Planner's Problem

$$\max_{\left\{\widetilde{c}_t, h_t, \widetilde{k}_{t+1}\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t N_t \left[\log \widetilde{c}_t + \psi \log (1 - h_t)\right]$$

subject to

$$\begin{aligned} \widetilde{k}_{t+1}\gamma\eta + \widetilde{c}_t &= (1-\delta)\,\widetilde{k}_t + z_t \widetilde{k}_t^{\alpha} h_t^{1-\alpha} &(4)\\ \widetilde{c}_t &\geq 0, h_t \in [0,1] \text{ and } \widetilde{k}_0 \text{ given} &(5)\\ z_t &= (1-\rho) + \rho z_{t-1} + \varepsilon_t, \varepsilon_t \tilde{} N\left(0, \sigma^2\right) &(6) \end{aligned}$$

- First order condition with respect to $\widetilde{c}_t$, $h_t, \widetilde{k}_{t+1}$ yield

$$\begin{aligned}
\frac{N_t}{\widetilde{c}_t} &= \lambda_t \\
\frac{N_t \psi}{1 - h_t} &= \lambda_t (1 - \alpha) z_t \widetilde{k}_t^{\alpha} h_t^{-\alpha} \\
\lambda_t \gamma \eta &= E_t \left[ \lambda_{t+1} \left( 1 - \delta + z_{t+1} \widetilde{k}_{t+1}^{\alpha} h_{t+1}^{-\alpha} \right) \right]
\end{aligned}$$

- Intratemporal optimality condition

$$\frac{\psi}{1-h_t} = \frac{(1-\alpha)\, z_t \widetilde{k}_t^{\alpha} h_t^{-\alpha}}{\widetilde{c}_t}$$

- Intertemporal optimality condition

$$\frac{1}{\widetilde{c}_t}\gamma = \beta E_t \left[ \frac{1}{\widetilde{c}_{t+1}} \left( 1 - \delta + z_{t+1} \widetilde{k}_{t+1}^{\alpha} h_{t+1}^{-\alpha} \right) \right]$$

- A balanced growth path is a situation where $\left(\widetilde{c}_t, \widetilde{k}_t, \widetilde{y}_t\right)$ are constant.

# The representative firm's problem in decentralized economy

$$\max_{K_t, H_t} z_t K_t^{\alpha} \left(\gamma^t H_t\right)^{1-\alpha} - w_t H_t - r_t K_t$$

- First order condition

$$\begin{aligned} w_t &= z_t (1-\alpha) (K_t/H_t)^{\alpha} \gamma^{t(1-\alpha)} = z_t (1-\alpha) \gamma^t \left(\widetilde{k}_t/h_t\right)^{\alpha} \\ r_t &= z_t \alpha K_t^{\alpha-1} \left(\gamma^t H_t\right)^{1-\alpha} = z_t \alpha \left(\widetilde{k}_t/h_t\right)^{\alpha-1} \end{aligned}$$

- Along BGP, $\widetilde{k}_t$ is constant. Therefore, $r_t$ is constant, while $w_t$ grows at a constant rate $\gamma$.

- Given our transformation of variables, the intratemporal and intertemporal optimality conditions in the original economy are equivalent to their counterparts in this stationary economy.

# Calibration

- One period in our model is one year.
- Parameters that have a time dimension: $\delta, \beta, \gamma, \eta$
- $\eta = 1.01$, the long run growth rate of working age population in the U.S.
- At the balanced growth path, $\gamma$ is equal to the growth rate of output per capita. $\gamma = 1.019$,which is the long run average growth rate of GDP per capita in the U.S.
- $\alpha = 0.33$ to match the average capital income share in the U.S.

- $\psi$ is set to target an average of one-third of their discretionary time working.

- To calibrate $\delta$, note that at balanced growth path, the law of motion for capital

$$\widetilde{k}\gamma\eta = (1-\delta)\,\widetilde{k} + \widetilde{i}$$

which implies

$$\delta = \frac{\widetilde{i}}{\widetilde{k}} + 1 - \gamma\eta = \frac{I}{K} + 1 - \gamma\eta$$

The long run average ratio $\frac{I}{K} = 0.076$, which yield an annual depreciation rate of 4.68% (or a quarterly rate of 1.17%).

- For $\beta$, Euler Equation at balanced growth path

$$\gamma = \beta \left( \alpha \frac{y}{k} + 1 - \delta \right)$$

- Capital output ratio is estimated to be 3. This yield an annual $\beta = 0.958$.

- $\sigma = 1.7\%$ and $\rho = 0.9$ to match the observed standard deviation and serial correlation of total factor productivity.

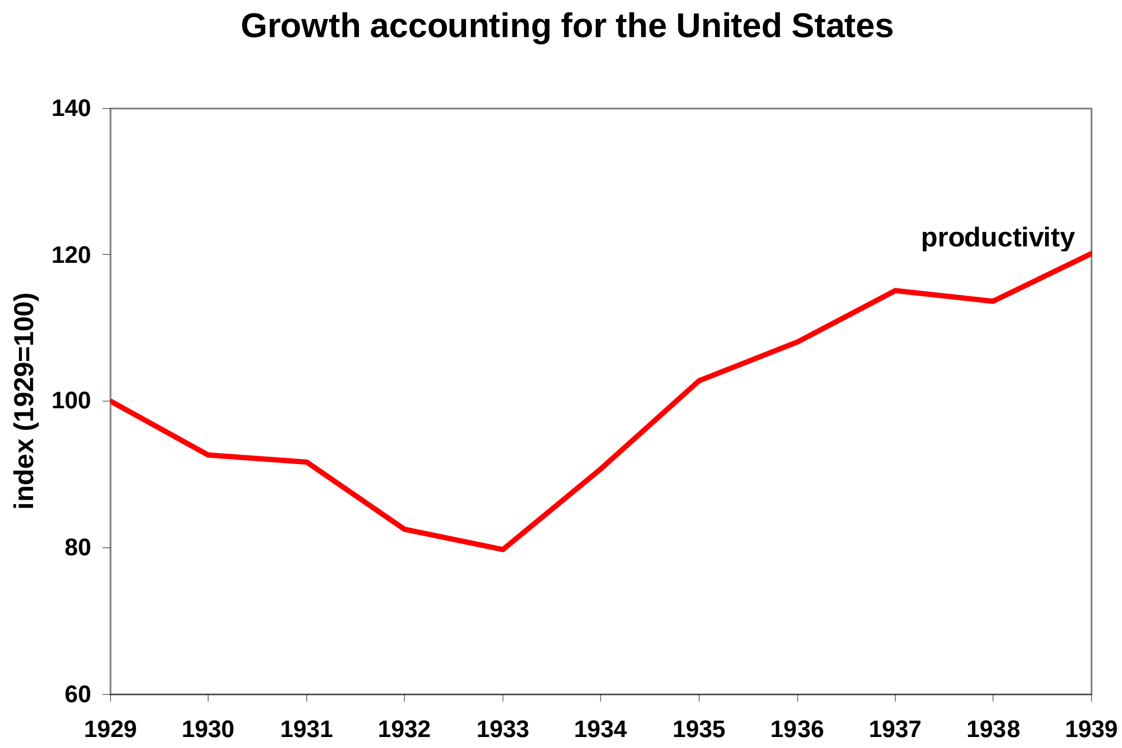
# Model's prediction

- Rewrite the social planner's problem as a dynamic programing problem.

- Solve the decision rule of this economy numerically and obtain $\widetilde{k}_{t+1} = g_k\left(\widetilde{k}_t, z_t\right), \widetilde{y}_t = g_y\left(\widetilde{k}_t, z_t\right), \widetilde{c}_t = g_c\left(\widetilde{k}_t, z_t\right), g_y\left(\widetilde{k}_t, z_t\right), h = g_h\left(\widetilde{k}_t, z_t\right)$.

- Assume capital stock in 1929 is equal to its steady state value.

- Feed in the sequence of observed levels of total factor productivity as measures of the technology shock.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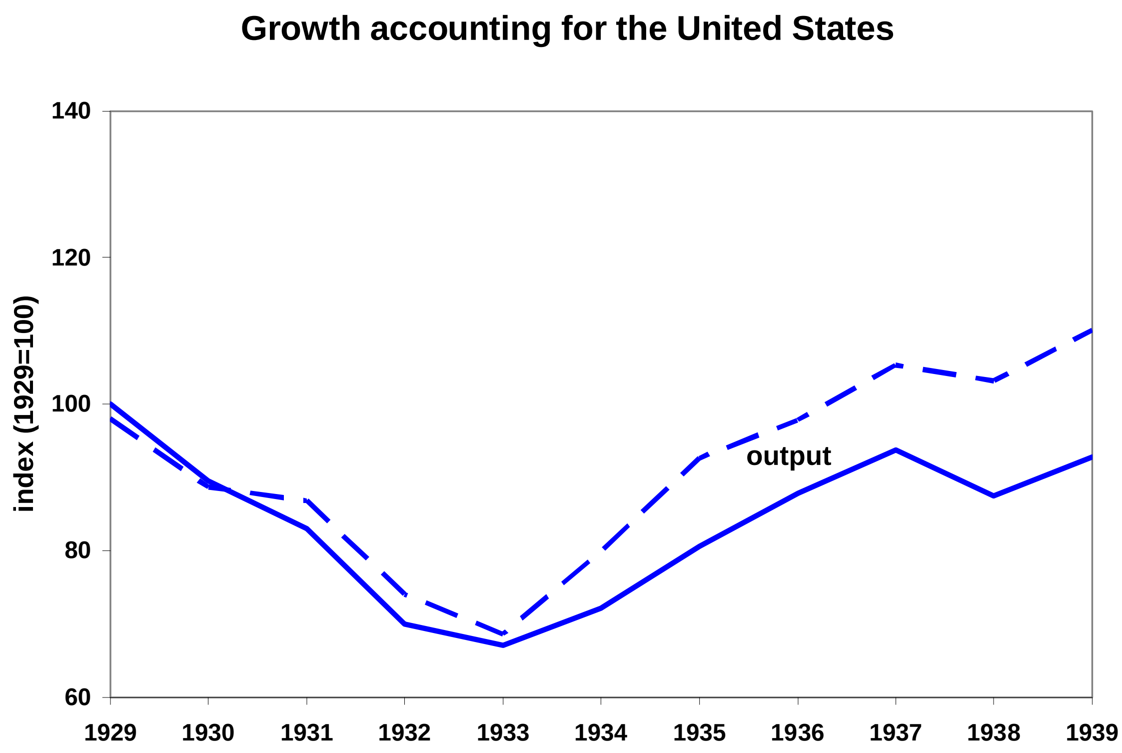
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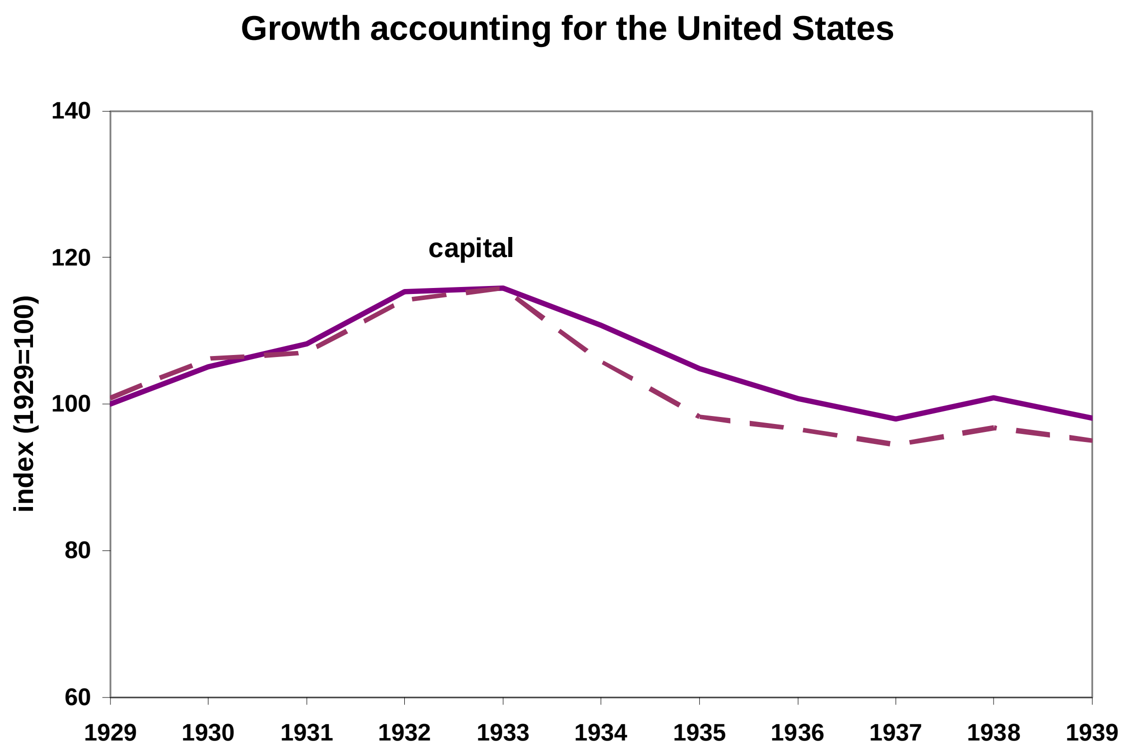
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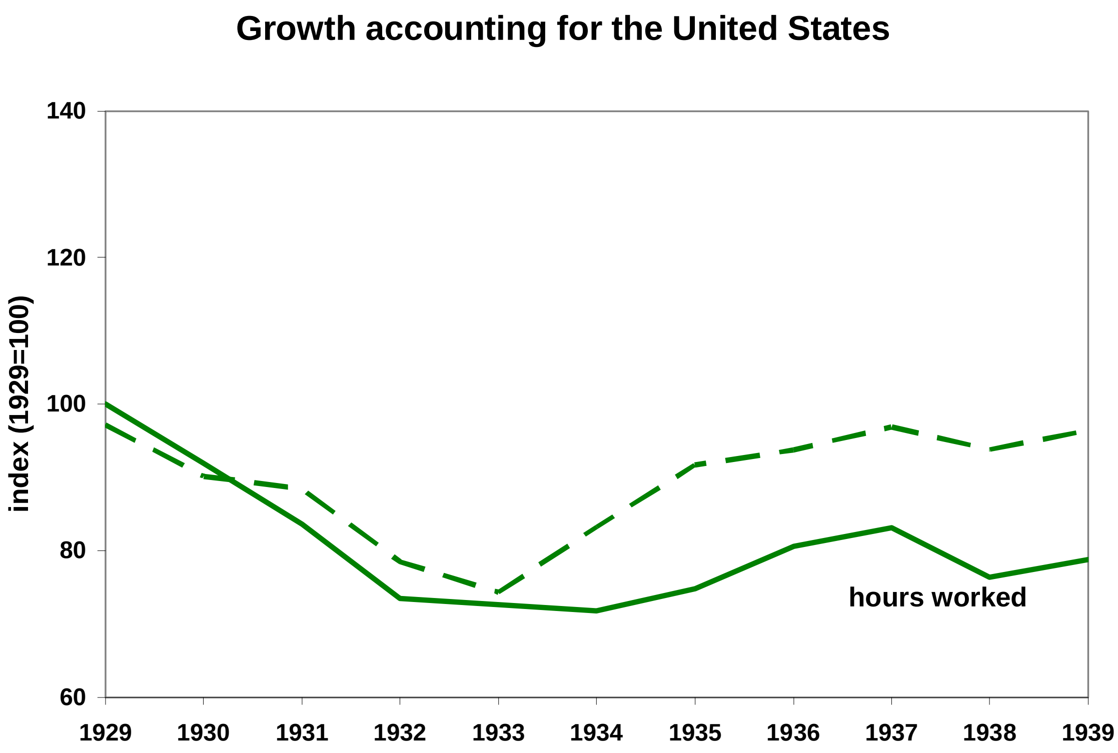
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# Conclusion

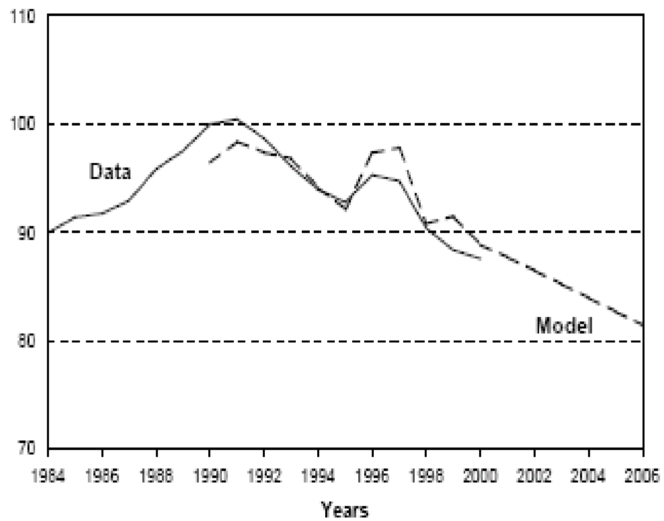
- A simple dynamic general equilibrium model that takes movements in the productivity factor as exogenous can explain most of the 1929-1933 downturn in the United States.
  - Keynesian analysis stresses declines in inputs of capital and labor as the causes of depressions.
- The model over predicts the increase in hours worked during the 1933-1939 recovery.
- Need for Further Study

- The decline in productivity during 1929-1933
- The failure of hours worked to recover 1933-1939.

# Other Applications of Neoclassical Growth Model

- The Japanese lost decade
- The Japanese saving rate
- The U.S. saving rate

**Figure. 6: Detrended real GNP per working-age person (1990=100**




Source: Hayashi and Prescott (2003)

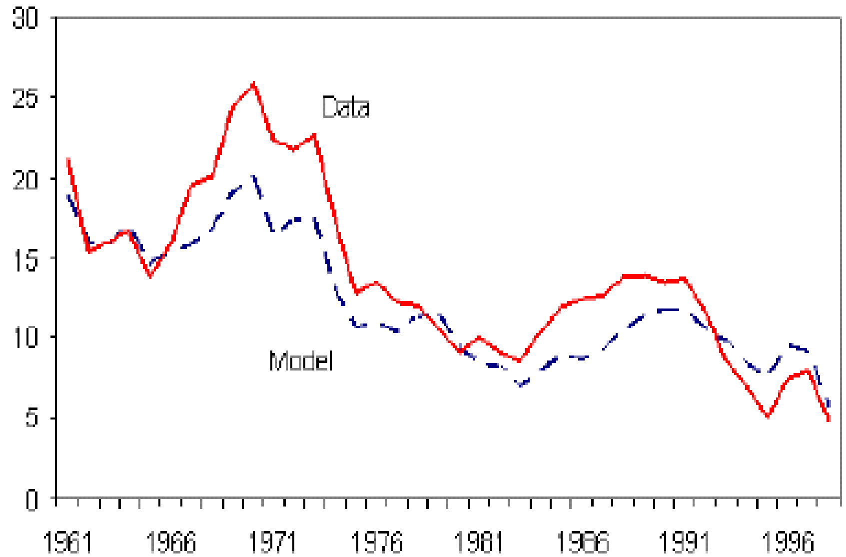



Figure 3: Saving Rate: Data and the Infinite Horizon Model

Source: Chen, Imrohoroglu and Imrohoroglu (2006)