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Exam
Advanced Macroeconomic Theory ECON 5300

Good Luck!

1 OLG Model

[60 points] Consider the two-period OLG economy discussed in the lecture with positive population growth, i.e., $n > 0$ and positive technological growth rate $g > 0$. Denote the depreciation rate as δ. The household's instantaneous utility function is CRRA: $U(C) = C^(1-\sigma)/(1-\sigma)$ and time discount rate is ρ, i.e., the lifetime utility is

$$\frac{C_y^{1-\sigma}}{1-\sigma} + \frac{1}{1+\rho}\frac{C_o^{1-\sigma}}{1-\sigma}.$$

The production function of the firm is Cobb-Douglas: $F(K, L) = K^{\alpha}(AL)^{1-\alpha}$.

1. [10 marks] Write down the household's problem and solve for household savings, as a function of wage and interest rate. How do savings respond to an increase of interest rate? Explain the answer.

2. Derive the dynamic equations that characterize aggregate capital K and capital per efficient unit of labour $k = K/(AL)$, where A is the technology level and L is population. An implicit equation is sufficient.

3. Suppose that the economy is on its balanced growth path before period t_0. Unexpectedly, at the beginning of period t_0, the population jumps to a higher level $L' > L(t_0)$ while population growth rate afterwards decreases to $n' < n$ and stays at the low level forever. How do aggregate output Y and output per efficient unit of labour $y = Y/(AL)$ from t_0 on respond, respectively? How does the long-run y compare to the initial y at period t_0? Explain your answers.
   ANS: Y at time t_0 jumps up while y drops. In the long run, y is higher than initial y at period t_0.

4. Suppose $\sigma = 1, n = g = 0$, and the economy is on its balanced growth path before period t_0. Unexpectedly, at the beginning of period t_0, the production function changes, α drops to a lower level $\alpha' < \alpha$ and stays at the new level forever. How does the long-run level of k change? How does k converges to the long-run level? Explain your answers.

2 Value Function

[40 points] Consider the following decentralised economy where the representative agent makes per capita consumption c_t and per capita labour supply l_t decisions,

$$max_{c_t,l_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t)$$

subject to

$$a_{t+1} = r_t a_t + w_t l_t - c_t,$$

where $u(.,.)$ is a per period utility function that satisfied the Inada conditions, $\beta \in [0, 1]$ denotes the discount factor, a_t the level of assets, r_t the real return to asset holdings and w_t the hourly wage. Assume that all markets operate under perfect competition and aggregate production is given by $Y_t = z_t K_t^{\alpha} N_t^{1-\alpha}$, where K_t denotes the stock of capital and N_t the total labour supply. Note that this formulation implies that per capita production is given by $y_t = z_t k_t^{\alpha}$, where $\alpha \in [0, 1]$ and z_t is a random technology shock that is known at the beginning of period t and E_0 is the expectation at time $t = 0$ based on the distribution of z. Business cycles are generated by z_t.

1. Using Bellman's principle of optimality write the above optimization problem in terms of its value function, $V(a_t, z_t)$.

2. From the derived Bellman equation and using the envelope theorem, show that the first-order condition that describe the optimal choices of consumption and leisure are given by

$$u_1(c_t, 1 - l_t) = \beta E_t[u_1(c_{t+1}, 1 - l_{t+1}) r_{t+1} | z_t]$$
$$u_2(c_t, 1 - l_t) = u_1(c_t, 1 - l_t) w_t,$$

   where $u_1(.)$ refers to the first derivative of u with respect to c and $u_2(.)$ refers to the first derivative of u with respect to l. Explain the economic intuition behind these equations.

3. Consider the case in which $ln(c_t) + b\frac{(1-l_t)^{(1-\gamma)}}{(1-\gamma)}$, where $\gamma > 0$ the elasticity of intertemporal substitution and $b > 0$. Using the Euler equation for consumption derived earlier, show that the savings rate is constant.