Econ 5315
Spring 2007
Chen

Problem Set 2
Due March 9, 2007

1. **Negishi Method**
Consider the simple pure exchange economy with three people $i = 1, 2, 3$ in the economy that live forever . each consumer values sequences of consumption $\left\{c_t^i\right\}_{t=0}^{\infty}$ according to

$$u_i\left(c^i\right) = \sum_{t=0}^{\infty} \beta_i^t \log c_t^i$$

with $0 < \beta_1 < \beta_2 < \beta_3 < 1$, and the deterministic endowment stream as

$$\begin{aligned} e_t^1 &= 1 \text{ for all } t \geqslant 0 \\ e_t^2 &= 1 \text{ for all } t \geqslant 0 \\ e_t^3 &= 2 \text{ for all } t \geqslant 0 \end{aligned}$$

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(a) Solve the Pareto problem and find the AD equilibrium with Negishi Method

(b) Assume that $\beta_1 = 0.995$, $\beta_2 = .997$, $\beta_3 = .998$. Calculate the equilibrium allocation. Interpret your results.

(c) Define the sequential market equilibrium for this economy. Find the Non-Ponzi condition, and use it, together with zero initial asset holdings for each agent, to derive the AD budget constraint. Interpret your result.

(d) Assume that the initial asset holdings are zero and find the SM equilibrium for this economy. Especially, find the transversality condition for this problem, Show that the No-Ponzi condition is met if transversality condition is met.

(e) Prove that the AD budget constraint implies the sequential budget constraints and the No-Ponzi Condition.

2 **Chatterjee meets labor supply**

Consider the following version of the model in Chatterjee (1994), extended to include labor supply.

There are I different types of infinitely-lived agents. Each agent $i \in I$ solves the following problem

$$\begin{aligned} & \max_{\{c_t,\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(c_{it}, h_{it}\right) \\ & s.t. \\ c_{it} + a_{i,t+1} &= \left(1 + r_t\right) a_{it} + w_t h_{it} \\ a_{i,t+1} &\geqslant 0 \end{aligned}$$

where w_t and r_t are the wage rate and the interest rate, respectively. hence, each agent makes a leisure-consumption trade-off where agent i decided how much to consume each period and how many hours to work, h_{it} at the economy-wide wage rate w_t times the agent's individual-specific efficiency units $\varepsilon_i > 0$. The average efficiency unit is equal to unity:

$$\frac{1}{I} \sum_{i=1}^{I} \varepsilon_i = 1$$

the economy is closed and capital depreciates at rate δ. The production side consists of a standard Cobb-Douglas production function taking aggregate labor N_t and capital K_t as inputs:

$$Y_t = F\left(K_t, N_t\right)$$

where aggregate labor is the total supplied efficient units, $N_t = \sum_{i=1}^{I} \varepsilon_i h_{it}$

(a) Write down the conditions for a competitive equilibrium.

(b) Assume that the utility function is given by

$$u\left(c, h\right) = \frac{\left(c^{\chi}\left(1-h\right)^{1-\chi}\right)^{1-\sigma}}{1-\sigma}$$

Show that aggregation holds, i.e. that the distribution of financial wealth and efficiency units have no impact on the dynamics of aggregate variables.

(c) Suppose the aggregate economy starts out relatively poor (i.e. with the aggregate capital stock). What are the dynamics of inequality in total wealth. Interpret your results.

(d) Assume that preference take the form

$$u(c,h) = v\left(\chi_0 + c^{\chi_1}(1-h)^{1-\chi_1}\right)$$

where the function v is homothetic (e.g. $CRRA$ or $CARA$). What condition must we impose on χ_0 and χ_1 in order to get the aggregation hold. What condition must we impose on χ_0 and χ_1 in order to get the property that individual consumption is a constant fraction of aggregate consumption across time.

(e) Given the preference in question (d), discuss the impact of χ_0 on the evolution of consumption inequality and wealth inequality over time, assuming that the economy start out below the steady state. Explain your results.