Econ 5315
Spring 2007
Chen

Problem Set 3
Due March 30th , 2007

1. **More on PILCH**
Consider the standard PILCH framework as discussed in the class. Agents are infinitely-lived. The period utility is quadratic. Assume $1 + r = 1 + \rho$ so that consumption follows the Martingale property.

   (a) Following Storesletten, Telmer and Yaron (1999), Suppose the individual income process is the sum of a persistent component and a transitory component

$$\begin{aligned} y_t &= y_t^p + u_t \\ y_t^p &= \alpha + \rho y_{t-1}^p + v_t \end{aligned}$$

   where both u_t and v_t are uncorrelated *i.i.d.* random variables with $E_t u_{t+s} = E_t v_{t+s} = 0$ for $s > 0$. Derive the formula for consumption growth $(c_t - c_{t-1})$ as a function of v_t and u_t. How does an increase in ρ affect the consumption growth? Does α affect the consumption growth? Explain your results.

   (b) Now let $\rho = 1$ so that v_t is permanent shock to income. How does income growth rate $g_t = \frac{y_t}{y_{t-1}}$ affect the fluctuation of saving rate, that is $\frac{s_t}{y_t} - \frac{s_{t-1}}{y_{t-1}}$? ($\frac{s_t}{y_t} = 1 - \frac{c_t}{y_t}$). Explain your answer. Hint: you may discuss three cases: (1) g_t is anticipated at time $t - 1$, that is g_t reflects the life-cycle changes of individual income. (2) g_t is driven purely by permanent income shock v_t. (3) g_t is purely driven by a transitory income shock, u_t, assuming $u_{t-1} = 0$.

   (c) Now assume individual cannot distinguish permanent from transitory income shock and let income take the form

$$y_t = y_{t-1} + \varepsilon_t - \gamma \varepsilon_{t-1}$$

with $\gamma \in [0,1]$, ε_t is iid. Note that when $\gamma = 1$, ε_t is transitory shock, while ε_t is permanent shock when $\gamma = 0$. Derive consumption growth as a function of ε_t and γ? How does an increase in γ affect the response of consumption growth to ε_t? Explain your results.

(d) Suppose that the income process for agent i is

$$\begin{aligned} y_{it} &= y_{it}^p + u_{it} \\ y_{it}^p &= \alpha_i + y_{i,t-1}^p + v_{it} \end{aligned}$$

where α_i is an individual-specific expected growth in income. Show that in this case, we lose both the identification of the shocks and the testable restriction of the theory, using the method of Blundell and Preston (1998)

2. **A basic risk sharing example**

Consider an economy consisting of two agents, agent A and agent B. Both agents have CRRA preference

$$\max E \sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

Agents' endowments are as follows: Every odd period ($t = 1, 3, 5$), the endowments are $y_{At} = 2$ for agent A and $y_{Bt} = 0$ for agent B. every even period ($t = 2, 4, 6, ...$), the endowments are reversed. $y_{At} = 0$ for agents A and $y_{Bt} = 2$ for agent B. Agents can trade a risk-free bound that comes in zero net supply.

(a) Compute the competitive equilibrium prices and allocation.

(b) Why does aggregation hold, even though there is only one asset available (the risk-free bond)?