

UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Home assignment for Ph.D.-candidates: **ECON9315 – Recursive Macroeconomic Theory**

Handed out: Friday, May 15, 2009

To be delivered by: Saturday, May 16, 2009

Place of delivery: Electronically to tone.enger@econ.uio.no

Further instructions:

- The questions are in English, and you are asked to give your answers in English.
- The home-exam will be marked and the scale for the mark will be pass/fail.
- After completion, please send your paper to the address given above. The paper must not bear your name, but the individual examination number, which you can find in your studentweb. If you have difficulties with access to your studentweb, please call the Department office, phone 22855127 or Tone Enger, phone 22855115
- In addition, you must fill in the enclosed declaration

ECON 5315- Recursive Macroeconomic Theory

Final Exam, May 15th, 2009

Before you start, please read the following:

- You have 24 hours to finish this take-home exam.
- Please answer in English.
- Answer all questions and write brief and concise answers!
- Allocate time spent on each question wisely.
- Good style will not matter for grades, but please write clearly.
- Good luck!

1 Aiyagari Model (60%)

Consider the Aiyagari model. The economy is populated by a large number of agents with total measure one. An agent solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{aligned} c_t + a_{t+1} &\leq s_t w_t + (1 + r_t) a_t \\ c_t &\geq 0, a_{t+1} \geq 0 \\ a_0 &> 0 \text{ given} \end{aligned}$$

where r_t is the real interest rate, w_t is the real wage rate per efficiency labor. The labor productivity shock follow a first-order, N -state Markov process $\{s_1, s_2, \dots, s_N\}$ with transition probability $\pi(s_{t+1} = s' \mid s_t = s)$. There is a large number of agents who face the same shock process. The firm has access to a constant returns to scale production technology $F(K_t, N_t)$, where K_t is aggregate capital, which depreciate at a rate δ , and N_t is aggregate labor in efficiency units.

1. Write the Bellman Equation for this problem. (10%)
2. Define a stationary recursive competitive equilibrium for this economy, clearly distinguishing between individual and aggregate state variables. (10%)

3. What is the natural borrowing constraint? Can you think of a joint condition on the period-utility u and on the transition matrix π such that the natural borrowing constraint is never binding for the agent, even if agent were allowed to borrow? (10%)
4. Suppose that the agent does not face any borrowing constraint. Does the agent display precautionary saving behavior? Discuss. (10%)
5. Draw a graph for “mean asset holdings” under No Insurance (i.e. self-insurance only) and Full Insurance (i.e. $s_t = \bar{s}$ for all t , where \bar{s} is the average of labor productivity), under which the economy collapses to a representative household setup. Show in the graph the magnitude of precautionary saving in both partial equilibrium (i.e. for any given interest rate $r < \rho$) and general equilibrium. (10%)
6. What do you expect the model to perform in terms of wealth distribution, compared with the U.S. data? Briefly discuss. (10%)

2 Aggregation under Complete Market (40%)

Suppose there are two types of consumers distinguished by their initial endowment of capital. In particular, type-1 consumers (who comprise fraction θ of the economy) are richer than type-2 consumers (who comprise fraction $1 - \theta$ of the economy): each type-1 consumer is endowed with k_0^1 units of capital and each type-2 consumer is endowed with k_0^2 units of capital, where $k_0^1 > k_0^2$. The two types of consumers are identical in all other aspects. Each consumer takes prices as given (in particular, each consumer takes the aggregate, or total capital at the beginning of period 1 as given when making saving decision at period 0). The equilibrium condition is that the total saving of the two types of consumers in period 0 must equal the aggregate capital stock that consumers take as given when deciding how much to save.

Assume each consumer’s utility function take the form $u(c_0) + \beta u(c_1)$, with $u(c) = \log c$. The production technology available to firms is $y = k^\alpha n^{1-\alpha}$, with $0 < \alpha < 1$, where y is firm’s output and k and n are the services of capital and labor, respectively (Since consumers do not value leisure, we normalize each consumer’s labor endowment to 1, and $n = 1$ in each period)

1. Derive the equilibrium aggregate capital stock in period 1 as a function of primitives (i.e. the parameters α , β , θ , and the initial capital stock k_0^1 and k_0^2). (20%)
2. Use your result in question (1) to show that change in k_0^1 and k_0^2 (i.e. the initial distribution of capital) that keeps aggregate capital *constant* in period 0 has no effect either on equilibrium aggregate savings or equilibrium prices. This is a version of *aggregation theorem* for this economy. Holding

the total amount of capital in period 0 constant, the behavior of aggregate in this economy does not depend on the distribution of capital in period 0. Explain which conditions make aggregation hold in this economy and why. (15%)

3. Suppose that the utility function takes the form: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, where $\sigma > 0$. Does an aggregation theorem like the one described in question (2) hold in this economy? Explain why or why not. (5%)



Declaration

Please fill in this form and hand it in to the Department of Economics together with your home-exam.

I hereby declare that my **home-exam**, handed in for

PhD.-course in ECON9315 – Recursive Macroeconomic Theory

at Department of Economics, University of Oslo

1. Has not been used for exams at other educational institutions, in Norway or abroad.
2. Contains no quotations or extracts from written, printed or electronic sources without the source being referred to.
3. All references are listed in the bibliography
4. I am aware the contravention of these rules are a form of cheating, and against the rules of the University.

Oslo, date.....

Students signature: