1. How optimal income taxation is influenced by labor market considerations

2. Minimum wage with extensive-margin

3. Involuntary unemployment: Inability to work

4. Optimal income taxation and unemployment matching framework
How optimal income taxation is influenced by labor market considerations

Minimum wage with extensive-margin

Involuntary unemployment: Inability to work

Optimal income taxation and unemployment matching framework
Main reference: **Boadway and Tremblay (CESifo ES 2013)**

Outline:

- **Demand-side considerations in full-employment:**
  - minimum wages and occupational choice
  - endogenous wage rates

- **Various sources of unemployment:**
  - inability to work
  - long-term search unemployment
  - temporary search unemployment [SKIP]
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Marceau and Boadway (SJE 1994)

- **Intensive**-margin model with participation choice and endogenous wages
- Minimum wage $w_{\text{min}}$ induces firms to lay workers off in low-wage firms
- $w_{\text{min}}$ welfare improving as long as participation choice is binding
- Welfare improvement greater with unemployment insurance
- Argument against this approach: It assumes enforcement of $w_{\text{min}}$ (while government cannot observe $w$ with intensive margin model)
Lee and Saez (JPubE 2012)

- Extensive margin model so all wages observed
- 2 types of occupation,
- Participation decision due to different tastes for leisure \( \chi_i \), linear utility: \( u_i = x_i - \chi_i \)
- Production of a unique consumption good \( F(h_1, h_2) \) depends on the number of low-skilled workers \( h_1 \) and the number of high-skilled workers \( h_2 \). Assume CRS in production.
- Wages are endogenous owing to imperfect substitutability of skills in the production process
- \( w_i = \frac{\partial F}{\partial h_i} \) \( i = 1, 2 \). Assume \( w_1 < w_2 \) at equilibrium.
• Labor demand elasticity (for low-skilled workers) is finite, labor supply elasticity is positive

• $w_{\text{min}}$ causes involuntary unemployment

$\Rightarrow$

• Minimum wage desirable if (a) government wants to redistribute to low-skilled workers ($g_1 > 1$) and (b) rationing created by minimum wage is efficient

• That is, if layoffs assigned to those with highest preferences for leisure, $w_{\text{min}}$ is welfare improving.
Intuitively:

- There is a welfare gain for low-skilled workers (whose wage increases) who are highly valued by the government \((g_1 > g_2)\).
- Those losing their low-skilled job and shifting to no-work are those with zero surplus for having a low-skilled job \(\Rightarrow\) The welfare loss due to involuntary unemployment caused by the (small) minimum wage is second order and represented by the shaded triangle (exactly as in the standard Harberger deadweight burden analysis).
Source: Lee and Saez (JPubE 2012, p.741). The figure depicts the desirability of introducing a small minimum wage starting from the competitive equilibrium. A small minimum wage creates a first order transfer to low skilled workers from other factors and a second order welfare loss due to involuntary unemployment (under the key assumption of efficient rationing).
The earnings gain of low-skilled workers (the shaded rectangle on the figure) due to $w_{\text{min}}$ is entirely compensated by an earnings loss of high-skilled workers as long as the supply elasticity is positive (non-vertical supply curve) and the demand elasticity is finite (non-horizontal demand curve):

$$d\Pi = \sum_i \left[ \frac{\partial F_i}{\partial h_i} dh_i - w_i dh_i - h_i dw_i \right] = 0$$

$\iff h_1 dw_1 = -h_2 dw_2$ (from no profit condition $\Pi = 0 \Rightarrow d\Pi = 0$)

Remark: **This rationing** of available jobs (**layoffs assigned to those with highest preferences for leisure**) is a **strong requirement**
**MIRRELES MODEL**: decisions about how much to earn and whether to work in the hands of individuals \[\Rightarrow\] Voluntary unemployment

\[\neq\]

Unvoluntary unemployment, different classes of involuntary unemployment:

- People unable to work
- Long-term unemployed who are capable of working but unable to find a job
- Temporarily unemployed (uncertain event both in terms of likelihood and its duration)
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Some persons are unable to work, what is the implications for optimal tax-transfer schemes?

- Extreme case where such persons cannot be identified:
  Transfers to them must be based on self-identification or self-selection and will be restricted by IC. IC (strongly) limits transfers to non-working so that able people do not mimic them.

- Gvt can acquire some individual info to relax the IC; Tagging (Akerlof AER 1978)
  - Disability imperfectly observable
  - Signal/tag positively correlated with disability
Tagging of disabled, Parsons (JPubE 1996)

- 2 groups: untagged and tagged
- More disabled among untagged
- Within each group, transfers to those not working are restricted by IC
- Lump-sum transfer can be made between groups. E.g., under utilitarian preferences: SW improves by redistributing from untagged to tagged group until average marginal utility of the same is the same across groups.
- However, such lump-sum transfer creates more inequality:
  - untagged disabled (Type I errors) are made worse off
  - tagged able (Type II errors) are made better off

$\Rightarrow$ Greater aversion to inequality $\Rightarrow$ lower transfer from untagged to tagged, so the less the value of tagging.

In the limit, **under maximin: tagging is of no use.**
Tagging of disabled

The **value of tagging is reduced** by:

- Type I errors (false negatives): the less accurate is tagging, the less useful is tagging as a way of relaxing the ICC
- Horizontal equity grounds: tagging treats differently identical persons depending on whether or not they are tagged
- Stigma attached to being tagged (Jacquet and Van der Linden FinanzA 2006), stigma due to the shame of being identified as tagged, even if only the tagging administrator observes it or the public knowledge that some non-deserving persons might be receiving transfers may throw suspicion on all transfer recipients
- Low take-up rates
- Complexity = vehicle by which non-deserving applicants are discouraged from applying (Kleven and Kopczuk, AEJ: Econ.Pol 2011)
- Cost of monitoring, type I and type II errors, non takeup (Jacquet, SCW 2014)
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Set aside the inability to work and concentrate on involuntary unemployment by those able to work

Key distinction: short term unemployment versus long term unemployment

Policy response to short-term unemployment: at least partly unemployment insurance

Policy response to long-term unemployment: more redistributive in nature
Optimal income taxation and unemployment matching framework

Survey of:

- Hungerbühler, Lehmann, Parmentier, Van der Linden. (ReStud 2006),
- Hungerbühler, Lehmann, Parmentier, Van der Linden (PE 2008),
- Hungerbühler and Lehmann (JPubE 2009)
- Jacquet, Lehmann and Van der Linden. (SCW 2014)

These papers depart from the usual assumptions, in the optimal tax literature, that labor markets are perfectly competitive and wage equals the marginal productivity.
Optimal income taxation and unemployment matching framework

- Employment levels depend on participation decisions and on bargained wages.

- Unemployment-matching framework (Diamond-Mortenssen-Pissarides): for a given wage, employment increases with labor demand and labor supply.

- Matching frictions imply that not all individuals find a job and not all firms find a worker.
Unemployment-matching framework

- Firms with concave production function in need of workers of a given type post vacancies
- Total vacancies: $V$ determined by zero-expected profit condition of vacancies, given free entry
- Workers of a given type who are unemployed choose to look for a job
- Number of workers searching: $U$
- Number of positions filled determined by matching function $M(U, V)$. Very often Cobb-Douglas: $M(U, V) = U^\alpha V^{1-\alpha}$ (linear homogeneous)
Unemployment-matching framework

- Jobs are filled randomly
- Firm: Proba of filling a job: \( \pi(\theta) \equiv M(U, V) / V = m(1/\theta, 1) \)
  where \( \theta \): indicator of market tightness, \( \pi(\theta) \) decreasing in \( \theta \)
- Worker: Proba of finding a job \( M(V, U) / U = \theta \pi(\theta) \) which is increasing in \( \theta \)
- Wage determination process: Nash bargaining
- Wage is chosen to maximize the Nash product of the surpluses to worker and firms, \((w - T(w) - b)^{\rho} (a - w)^{1-\rho}\) where \( b \): transfer to unemployed, \( \rho \): relative bargaining power of workers.
- If \( \rho = \alpha \), this bargaining process is efficient in the sense that externalities of search are internalized, Hosios (ReStud 1990).
Hosios condition

- When a firm creates a vacancy
  ⇒ proba of workers finding a job ↑
  ⇒ proba of other firms finding a match ↓
- When a worker chooses to search
  ⇒ proba of firms finding a match ↑
  ⇒ proba of other workers finding a job ↓
- When **Hosios condition** satisfied, **these effects are offsetting**.
“Equivalently, Hosios condition implies that the share of the surplus captured by workers in the bargaining process reflects a worker’s relative productivity at generating matches, and similarly for firms, so that workers’ search effort and firms’ decisions to create vacancies are efficient.” (Boadway and Tremblay, 2013, p.123)
In an unemployment matching framework, a series of papers, beginning with Hungerbühler, Lehmann, Parmentier, Van der Linden (ReStud 2006), have explored the consequences of permanent involuntary unemployment for the structure of optimal income taxes.
Wages are affected by taxation: the **bargained wage** (i) decreases with marginal tax rate $T'$ and (ii) increases with tax level $T$.

(i) $T' \uparrow$ (without affecting $T$) $\Rightarrow$ $\downarrow$ pre-tax wage (intuitively, a rise in $T'$ implies that an increase in gross wage has a reduced impact on net wages while firm’s profit is unchanged. Therefore, it becomes less rewarding for workers to bargain aggressively, and gross wages fall) $\Rightarrow$ $\uparrow$ labor demand and $\downarrow$ unemployment. “**Wage-cum-labor-demand margin**”.
Wages are affected by taxation: the **bargained wage** (i) decreases with marginal tax rate $T'$ and (ii) increases with tax level $T$.

(ii) $\uparrow$ (average) tax level (without affecting $T'$) $\Rightarrow \uparrow$ pre-tax wage (intuitively, a rise in the tax level reduces worker’s ex post surplus (see later)) $\Rightarrow$ Workers claim higher wages
$\Rightarrow \downarrow$ labor demand and $\uparrow$ unemployment.
Deadweight losses of taxation due to responses along the \textit{wage-cum-labor-demand margin} (and not along the intensive labor supply margin like in Mirrleesian literature)
The government’s program

- The Government (Gvt) observes only whether an individual is employed or not, and if she is, at which wage.
- The Gvt does observe neither skills nor the recruiting processes. Hence, taxation is only a function of wages.
- No tax evasion, no side-payment.
- The Gvt maximizes a SWF (e.g. Bergson-Samuelson or maximin) subject to the Gvt’s budget constraint and the choices made by the agents.
Employed, unemployed and non-participants

People are risk-neutral and differ w.r.t.

- their skill \( a \sim f(a) \) on \([a_0, a_1]\), with \( 0 < a_0 < a_1 \leq +\infty \)

- *Skill-specific* labor markets (LM): A worker of skill \( a \) produces a units of output if and only if she is employed in a type-\( a \) job, otherwise her production is nil, i.e. perfect segmentation (more realistic than the polar one of a unique labor market for all skill levels)

- value \( \chi \) of being out of the labor force \( \chi \sim \) with conditional CDF \( G(., a) \).
In Hungerbühler, Lehmann, Parmentier, Van der Linden (ReStud 2006), all workers have the same value of leisure \( \Rightarrow \textbf{cutoff skill} \) level \( \tilde{a} \) such that workers participate iff \( a \geq \tilde{a} \).

This assumption is relaxed in subsequent papers.
Employed, unemployed and non-participants

People are risk-neutral and differ w.r.to

- Employed workers of skill $a$ get a (pre-tax) wage $w_a$ and a disposable income $c_a = w_a - T(w_a)$
- On skill-$a$ LM, only a fraction $L(a, w_a)$ of the $f(a)$ skill-$a$ individuals find a job.
- Employment probabilities are given by $l_a = L(a, w_a)$ on LM of skill $a$
- Unemployed and non-participants (i.e. unvoluntary and voluntary unemployed) get $b$ (job search not monitored by the Gvt)
- Unemployed get $b$ and non-participants get $b + \chi$
Firms

- Opening a vacancy costs $\kappa(a)$ which includes the investment in equipment and the screening of applicants.
- Vacancy cost increases less than proportionaly or decreases with the skill level $a$; $a\kappa(\alpha) / \kappa(a) \leq 1$
- A zero-profit condition determines the number of vacancies created by firms on each skill-specific LM
- Critically, while Gvt observes employment earnings, it cannot observe worker abilities. ⇒ Possibility that firms employing workers of one skill level can mimic the bargaining outcomes of others ⇒ Incentive constraint applied to wage bargains.
Optimal income taxation and unemployment matching framework

Timing (static version of the matching model)

1. The government sets the tax function $T(\cdot)$ and the level of the assistance benefit $b$.

2. On labor market specific to skill $a$: $V_a$ vacant jobs are created, $U_a \leq f(a)$ individuals search for a job (costs $\chi$).

3. Matching occurs. It determines the number $H(a, V_a, U_a)$ of jobs. Once matched, the firm and the worker negotiate the wage $w_a$.

4. Production, transfers and consumption occur.
Participation decisions

An individual of type \((a, \chi)\) participates iff:

\[
\ell_a (w_a - T (w_a)) + (1 - \ell_a) b \geq b + \chi
\]

\(x_a \overset{\text{def}}{=} w_a - T (w_a) - b\): ex-post surplus on LM \(a\).

\(N_a \overset{\text{def}}{=} \ell_a [w_a - T (w_a) - b]\): expected surplus of a participant of type \(a\).

An individual of type \((a, \chi)\) participates iff \(N_a \geq \chi\)

\(\Rightarrow\) Skill-specific participation rate equals \(G (a, N_a)\) where

\(G (a, x) \overset{\text{def}}{=} \Pr (\chi \leq x \mid a)\) is the conditional CDF.
Labor demand under matching frictions

The following assumptions:

Perfect segmentation of LM by skill.

*Free-entry* of vacancy on each skill specific LM.

and *constant returns to scale* Matching functions $H(a, ., .)$.

... defines a *labor demand/employment probability function* $\ell_a = L(a, w_a)$ where

$L(a, .)$ is decreasing in the wage $w_a$ on its own labor market

$L(a, .)$ does not depend on the wage $w_c$ in the other labor markets $c \neq a$

$L(a, .)$ does not depend on the number of participants $U_a$.

**Lemma:** One can retrieve the matching function $H(., ., .)$ and the vacancy costs $\kappa(.)$ from labor demand $L(., .)$.
Wage-setting

Wages are the outcome of a Nash bargain which leads, under Hosios condition*, to:
Wages $w_a$ solves, on each skill-specific labor market, the following wage setting objective

$$w_a = \arg \max_w L(a, w) \cdot \left( w - T(w) - b \right) \equiv N_a \text{ or } N(x, w, a) \quad (1)$$

where $N_a$ is the expected surplus of a participant of type $a$. The wage setting is increasing in disposable income $x$ (an employee’s welfare depends positively on the after-tax wage) and decreasing in the gross wage $w$ (a higher gross wage reduces firms’ profit and thus labor demand).

*Hosios (ReStud 1990) condition: the bargaining power of workers equals the elasticity of the matching function with respect to the stock of unemployment (see before)
Competitive Search Equilibrium of Moen (JPE 1997) or a skill-specific utilitarian monopoly union which selects the wage $w_a$ before firms decide about vacancy creation (Mortensen and Pissarides, 1999) give also (1), see proof in Lehmann, Parmentier and Van der Linden (JPubE 2011).
The worker’s surplus $x$ has to increase when the gross wage $w$ increases to keep the Nash product $N(.,..,a)$ unchanged. It can be shown that, for each pair $(w,x)$, the MRS

$$\frac{\partial x}{\partial w}\bigg|_{N_{a}(a,..)}$$

is a decreasing function of the type $a$. ⇒ Single-crossing condition.
Single-crossing condition
Wage-setting and Gvt’s problem

From (1) and $a \frac{\kappa(\alpha)}{\kappa(a)} \leq 1$ (in HLPV ReStud 2006) or $\frac{\partial^2 \log L}{\partial a \partial w}(a, w) > 0$ (the latter comes from assumption that wages increase in skill, in LPV JPubE 2011):

The maximized Nash product or expected participant’s surplus $N_a$ is increasing in skill $a$. 

Laurence Jacquet (THEMA, OFS)
Optimal income taxation and unemployment matching framework

Taxation principle applies in a matching framework

The set of allocations induced by a tax system $T(.)$ through the wage setting equation $\max_{w_a} L(a, w) \cdot (w - T(w) - b) \equiv N_a$ corresponds to the set of incentive-compatible allocations $\{w_a, x_a, N_a\}_{a \in [a_0, a_1]}$ that verify:

$$\forall (a, b) \in [a_0, a_1]^2 : N(x_a, w_a, a) \geq N(x_a, w_a, b)$$

This condition expresses that a worker-firm pair of type $a$ chooses the bundle $(w_a, x_a)$ designed for her, rather than any other bundle $(w_b, c_b)$ designed for worker-firm pairs of any other type $b$. 
The government’s problem [Skip]

An allocation \( a \mapsto (w_a; x_a = w_a - T(w_a) - b) \) is incentive-compatible iff:

\[
\forall (a, c) \quad N_a = L(a, w_a) \cdot x_a \geq L(a, w_c) x_c \quad \text{(IC)}
\]

**Taxation Principle** in a matching framework:
For any \( w \), \( L(a, w_a) \cdot x_a \geq L(a, w) (w - T(w) + b) \), so for \( w = w_c \), one obtains Incentive Compatibility (IC).
Proof. Assume that \( a \mapsto (w_a; x_a = w_a - T(w_a) - b) \) verifies IC. Take \( w \in \mathbb{R}^+ \). Either there is no \( a \) such that \( w = w_a \), in which case define \( T(w) = +\infty \).

Or take \( T(w) = w_a - x_a - b \) (the definition is unambiguous if \( w = w_a = w_c \) for \( a \neq c \)). In such a case, one must have \( x_a = x_c \) otherwise IC would be violated).

For all \( a \), such a tax function leads to \( w_a \) maximizes
\[
L(a, w) (w - T(w) - b)
\]
The government’s problem

Because the strict single-crossing condition holds, incentive constraints are reduced to

- a **differential (envelope) equation**

\[ \dot{N}_a = N_a \frac{\partial \log L}{\partial a} (a, w_a) \quad (IC_1) \]

and

- a **monotonicity constraint**: \( a \mapsto w_a \) is non decreasing \((IC_2)\).

usual proof: see e.g. Salanié (2005)
The government’s problem

The budget constraint gives $b$ through:

$$b = \int_{a_0}^{a_1} (T(w_a) + b) \ L(a, w_a) \ G(a, \Sigma_a) \ f(a) \ da - E$$

where $E \geq 0$ is an exogenous amount of public expenditures.
Maximin optimum (in LPV 2011)

\[
\max_{(w_a, \Sigma_a) \in [a_0, a_1]} \quad b = \int_{a_0}^{a_1} \left[ w_a L(a, w_a) - \Sigma_a \right] G(a, \Sigma_a) f(a) \, da - E
\]

s.t.  \quad \frac{\dot{\Sigma}_a}{\Sigma_a} = \frac{\partial \log L}{\partial a} (a, w_a)

\( a \mapsto w_a \) is nondecreasing
Maximin optimum [Skip]

The FOC at skill level \( a \) is

\[
\frac{1 - \eta (w_a)}{\eta (w_a)} \cdot \frac{\varepsilon_a}{\alpha_a} \cdot w_a \cdot a \cdot h_a = Z_a \quad Z_{a0} = 0
\]

\[
Z_a = \int_a^{a_1} [w_t - T (w_t) - b - \pi_t (T (w_t) + b)] h_t \cdot dt
\]

where:

\[
\varepsilon_a \overset{\text{def}}{=} \frac{\partial \log w_a}{\partial \log \eta} \quad \alpha_a \overset{\text{def}}{=} \frac{\partial \log w_a}{\partial \log a} \quad \pi_a \overset{\text{def}}{=} \frac{\Sigma_a \cdot g (a, \Sigma_a)}{G (a, \Sigma_a)}
\]

\[
h_t \overset{\text{def}}{=} L(t, w_t) \cdot G(t, \Sigma_t) \cdot f(t)
\]
Maximin optimum [Skip]

For individuals of skill $t$ above $a$

**Mechanical:** those who are working pay a higher level of tax

$$\Delta T_w(t) = \Delta x_t = -x_t \cdot \Delta \eta \times \frac{\delta w}{w_a}$$

**Participation:** a lower number of them enter the labor market

$$\Delta h_t = \pi_t \cdot h_t \cdot \Delta x_t / x_t$$

Each of them generating tax revenues $T(w_t) + b$

Aggregate mechanical and participation effects

$$-Z_a \cdot \Delta \eta \times \frac{\delta w}{w_a} =$$

$$\int_a^{a_1} \left[ -(w_t - T(w_t) - b) + \pi_t \cdot (T(w_t) - b) \right] h_t \cdot dt \cdot \Delta \eta \times \frac{\delta w}{w_a}$$
Maximin optimum [Skip]

Participants of skill $t$ within $[a - \delta a, a]$
An interval of the skill distribution of size $\delta a = a \frac{\delta w}{\alpha_a w_a}$

$$\frac{a}{\alpha_a} \cdot G(a, N_a) \cdot f(a) \cdot \frac{\delta w}{w_a}$$

Wages change by

$$\Delta w_a = \frac{w_a}{\eta(w_a)} \cdot \varepsilon_a \cdot \Delta \eta$$

Changing tax revenues per participant

$$L(w_a) (T(w_a) + b) = L(w_a) \cdot w_a - \Sigma_a$$

by

$$(1 - \eta(w_a)) \cdot L_a \cdot \Delta w_a$$

Wage response effect:

$$\frac{1 - \eta(w_a)}{\eta(w_a)} \cdot \frac{\varepsilon_a}{\alpha_a} \cdot w_a \cdot h_a \cdot \Delta \eta \times \frac{\delta w}{w_a}$$
If everywhere along the Maximin optimum one has \( \dot{\pi}_a < 0 \), then compared to the *laissez faire*, Skill-specific wages and unemployment rates are distorted downwards, except at the bottom and at the top. Compared to the *laissez faire*, the participation rates are distorted downwards.

\[ w \rightarrow T(w) / w \text{ is increasing, } T'(w) > 0 \text{ everywhere and } -T(w_a) < b. \]

In particular \( T'(w_1) = \frac{T(w_1)+b}{w_1} > 0 \) and \( T'(w_0) = \frac{T(w_0)+b}{w_0} > 0 \) and \( -T(w_{a_0}) < b \).
That is, under maximin (where the involuntary unemployed are the least well-off persons):

- Marginal tax rate positive everywhere.
- Marginal tax rate tends to be higher than in a competitive labor market setting with no unemployment.
- Participation/Employment tax rates are positive including at the bottom (no EITC).
- If the elasticity of participation falls with skill level, average tax rate is increasing in earnings.
Optimal tax formulae with Bergson-Samuelson SWF

The FOC at skill level \( a \) is

\[
\left( 1 - \frac{\eta (.)}{\eta (.)} \right) \frac{w_a}{\eta (.)} - \frac{\Phi (w_a - T (.) - \Phi (b) - x_a \cdot \Phi' (w_a - T (.) \right)}{\lambda} = \alpha_a \cdot \frac{Z_a}{\varepsilon_a} \cdot \frac{Z_a}{a \cdot h_a} \quad Z_{a_0} = 0
\]

\[
Z_a = \int_{a}^{a_1} \left\{ \left( 1 - \frac{\Phi' (w_t - T (.) \right)}{\lambda} \right) x_t - \pi_t \left[ T (.) + b + \Xi_t \right] \right\} h_t \ dt
\]

\[
\Xi_t = \frac{\ell_t \cdot \Phi (w_t - T (.) + (1 - \ell_t) \Phi (b) - \Phi (b + \Sigma_t)}{\lambda \cdot \ell_t}
\]

\[
\lambda = \int_{a_0}^{a_1} \left\{ \ell_a G (.) \Phi' (w_a - T (.) + (1 - \ell_a) G (.) \Phi' (b)
\right\} f (a) \ da + \int_{\Sigma_a}^{+\infty} \Phi' (b + \chi) g (a, \chi) \ d\chi
3 new terms:

1. \(-\frac{\Phi(w_a - T(w_a)) - \Phi(b) - x_a \cdot \Phi'(w_a - T(w_a))}{\lambda}\): a “within-skill” motive of redistribution: decreasing \(w_a\) reduces the size \(x_a\) and the occurrence of the inequality between employed and unemployed of the same skill (wage response effect).

2. \(\int_a^{a_1} - \frac{\Phi'(w_t - T(w_t))}{\lambda} x_t \cdot h_t \cdot dt\): because welfare of employed workers is now socially valued, the “between-skill” motive of redistribution is reduced. (mechanical effect)
3. \[ \int_a^{a_1} \mathbb{E}_t h_t \, dt = \int_a^{a_1} \frac{\ell_t \cdot \Phi(w_t - T(w_t)) + (1 - \ell_t) \Phi(b) - \Phi(b + \Sigma_t)}{\lambda \cdot \ell_t} h_t \, dt \] raising participation generates inequality for new participants (participation effect).
Analytical results

- $T'(w_{a_1}) > 0$ even when the skill distribution is bounded.
- Unemployment and wages are downward distorted at the top skill $a_1$.
- If no bunching, Unemployment and wages are downward distorted at the bottom skill $a_0$.
- **Simulations are required.**
Alternative wage bargaining and shutting down intensive margin

Jacquet, Lehmann and Van der Linden (SCW 2011)

- precludes the possibility of one skill level mimicking another by choosing the same wage $\Rightarrow$ Gvt can effectively observe wage bargaining in each group (although it is precluded from setting a tax based on skills instead of earnings) $\Rightarrow$ no IC.

- Leontief (Kalai) bargaining rather than Nash bargaining: $\max \min \left\{ \frac{w - T(w) - b}{\rho}, \frac{a - w}{1 - \rho} \right\}$, where $\rho$ reflects the bargaining strength of workers.

- When a match occurs: the sum of firm’s surplus $(a - w_a)$ and worker’s surplus $(w_a - \tau_a)$ is exogenously shared between the firm (fraction $1 - \rho(a)$) and the worker ($\rho(a)$). $\Rightarrow$ No effect of marginal tax rates on bargained wages.

- $\Rightarrow$ The shares of the surplus accruing to workers and firms are fixed

  $w(a) = \rho(a) a + (1 - \rho(a)) (T(w(a)) + b)$: The equilibrium wage is increasing in the level of tax.
An increase in tax, on a given skill level:

- reduces labor demand (since wage $\uparrow$)
- reduces labor supply (since participation $\downarrow$)
Alternative wage bargaining and shutting down intensive margin

Optimal (participation) employment tax:

\[
\frac{\tau(a)}{a - \tau(a)} = \frac{1 - g(a) \cdot \rho(a) \left( 1 + \eta^D(a) \right)}{\rho(a) (\eta^P(a) + \eta^D(a) + \eta^P(a) \eta^D(a))}
\]

where \( \eta^P(a) \): elasticity of participation of type \( a \)-workers
\( \eta^D(a) \): elasticity of labor demand w.r.t. surplus \( a - w(w) \).

Clear that there are **both demand and supply influences at work** in labor matching models.
A change in the employment tax affects employment through 3 channels

A larger employment tax $\tau_a \Rightarrow$

(1) reduces the number of job-seekers (labor supply response along extensive margin)

(2) increases the bargained wage $\Rightarrow$ reduces the labor demand (i.e. the number of job vacancies)

(3) This reduction of labor demand reduces the proba of finding a job hence the return of participation $\Rightarrow$ reduces the number of job-seekers (labor supply response). This complementarity effect appears through $\eta_a^P \eta_a^D$. 
The optimal employment taxes depend on the complementarity between labor supply and demand.

\[
\frac{\tau(a)}{w(a) - \tau(a)} = \frac{1 - g(a) \rho(a) (1 + \eta_D^a)}{\eta_D^a + \eta_P^a + \eta_P^a \eta_D^a} = \frac{1 - g(a) \rho(a) (1 + \eta_D^a)}{\rho(a) \eta_G^a}
\]

where \( \eta_G^a \equiv \eta_D^a + \eta_P^a + \eta_P^a \eta_D^a \) (general elasticity of employment).

At the denominator:

- When the participation elasticity \( \eta_P^a \) is larger, optimal to reduce \( \tau_a \) (like in the pure extensive model)
- When the labor demand elasticity \( \eta_D^a \) is larger, optimal to reduce \( \tau_a \)
- \( \eta_P^a \eta_D^a \): Complementarity between labor demand and supply that is a key insight in the unemployment matching theory:
  - When labor demand \( \downarrow \Rightarrow \) some agents stop searching for a job.
The optimal employment tax: The numerator

Intuition behind the numerator: $d\tau_a > 0 \Rightarrow$

- The expected surplus awarded to a type-$a$ agent ↓ by $\rho(a)$ units which the gvt values at rate $g(a)$.
- Labor demand also ↓ (since $w(a) \uparrow$) hence proba of finding a $a$-job ↓ that reduces the expected surplus. This is captured by the elasticity term $-\rho(a)\eta^D_a$ which the gvt values at rate $g(a)$. 
Model with labor demand and supply responses and wage and participation functions given in a reduced form way

Kroft, Kucko, Lehmann, and Schmieder (IZA WP 2016)

- Sufficient statistics approach
- Wages and the probability of finding a job (and, hence, employment) are endogenous to the tax system; extension of Saez (QJE 2002)
- **Wage and participation functions are given in a reduced form way**: they use reduced-forms to describe the macro responses (i.e. general equilibrium) of (gross and net) wages and conditional employment probabilities to taxation.
- Government’s objective depends only on expected utility
Case 1. No cross-effects (i.e. cross-elasticities) between sectors (taxes, wage, employment and participation level on one sector do not affect another sector):

\[
\frac{T_j + b}{c_j - b} = \frac{1 - g_j \frac{\pi_j}{\pi^m_j}}{\eta^G_j}
\]

where: \( \pi^m_j \): micro participation elasticity in the hypothetical case where tax changes do not affect gross wages and conditional employment probabilities (measures the percentage of employed workers in \( i \) who leave the labor force when the tax liability is increased by 1 percent, holding wages and the conditional employment probabilities fixed).

\( \pi_j \): macro participation elasticity

\( \eta^G_j \): macro employment elasticity (or general employment elasticity as above)

Macro responses do allow for certain equilibrium adjustment mechanisms, Micro responses do not.
Optimal tax formula when no cross-effects

- An EITC (on the working poor $j$) is optimal iff $g_j \frac{\pi_j}{\pi_m} > 1 \iff g_j > \frac{\pi^m_j}{\pi_j}$

The macro response to taxation ($\pi_j$) will be larger (than $\pi^m_j$) if a reduction in taxes leads to more job creations (i.e., mostly a labor demand channel) rather than a decrease in wages overall (i.e., an increase in labor supply).

$\frac{\pi_j}{\pi^m_j} \downarrow$ if a decrease in taxes mostly leads to an increase in labor demand.
Kroft, Kucko, Lehmann, and Schmieder use CPS and ORG data on single women (age 18-55),

An occupation/labor market = a group of workers within a state, education, and year.

They estimate the micro and macro responses to taxes of participation and employment within each of these labor markets, using policy variation in tax liabilities stemming from the U.S. tax and transfer system for 1984-2011.

Variations across Time State identify macroeconomic responses.
Variations across Time State #Kids identify microeconomic ones.
They simulate the optimal tax system

They empirically find that macro participation < micro participation, making EITC less desirable than in Saez (QJE 2002).

Remark regarding assumptions: no cross-effects between labor markets and the effect of \( b \) and \( T_i \) are assumed identical.
Reference

All references are recommended but references preceded by asteriks are compulsory. Main reference:
and all cited works in it.