EXAM
ANSWER KEY
STV1266

Introduction to Game-theoretic Models of War
Spring 2025

9:00
25 April 2025

Instructions: You have four hours to take the exam. Please allocate your time carefully. Answer all the questions.

Question 1 (20%)

Consider Table 1 and answer the following questions.

1. Find the game's Nash equilibrium (or equilibria) in pure strategies.

   The pure strategy Nash equilibrium is {(Limited, Limited), (Total, Total)}.

2. Does the game contain another Nash equilibrium in mixed strategies, yes or no? If yes, what is it? If no, why not?

   Yes, there is a mixed strategy Nash Equilibrium.

   Let p = Pr(Country 2 Limited) and q = Pr(Country 1 Limited).

   $Euc1(Limited) = p \cdot 6 + (1 - p) \cdot 0 = 6p$

   $Euc1(Total) = p \cdot 4 + (1 - p) \cdot 2 = 2 + 2p$

   Country 1 prefers to attack when:

   $6p \geq 2 + 2p$

   $4p \geq 2$

   $\Rightarrow p \geq 2/4$

   $\Rightarrow p \geq 1/2$

   By parallel arguments (due to the symmetry of payoffs), we find $q \geq 1/2$. The mixed strategy Nash
   Equilibrium is given by Country 1 playing Limited with frequency 1/2 and Total with frequency 1/2 and
   Country 2 playing Limited with frequency 1/2 and Total with frequency 1/2 .

3. Which outcome(s) are Pareto optimal?

   There is one Pareto optimal outcome: (Limited, Limited)}.

4. Which outcome(s) are suboptimal?

   There are three Pareto suboptimal outcome: {(Limited, Total), (Total, Limited), (Total, Total)}.

5. In a few sentences, discuss how this game and its results connect to coordination games and the intensity of conflict (e.g., Britain and Germany's naval warfare during World War I).

Free response. Points for discussing the best response to coordinate with the move of the opponent. Points if the exam mentions limited war is welfare-maximizing individually and collectively. Also points if the exam mentions historical context, norms, and preemptive war.

		Country 2	
		Limited	Total
Country 1	Limited	6,6	0,4
	Total	4,0	2,2

Figure 1: Question 1 — Intensity of Fighting Normal Form

The Figure is a 2 by 2 matrix. Country 1 is on the row and can choose between Limited war and Total war. Country2 is on the columns with choices of Limited and Total. If both countries choose Limited, they both get a payoff of 6 (or 6, 6). If Country 1 chooses Limited and Country 2 chooses Total, Country 1 gets 0 and Country 2 gets 4. If Country 1 chooses Total and Country 2 chooses Limited, Country 1 gets 4 and Country 2 gets 0. If they both choose Total, they both get 2 (or 2, 2).

Question 2 (30%)

Figure 2 shows a simple preventive-war model, where D is a declining state and R is a rising state.

a. Is the game in Figure 2 static or dynamic? Explain your answer.
The game in Figure 2 is dynamic, since D moves first, and R moves second.

b. What is a subgame-perfect equilibrium?
A subgame-perfect equilibrium (SPE) is a pair (set) of strategies that are best responses to each other for all subgames.

c. Find the game's subgame-perfect equilibrium when $c < 1$ (think of c as R's cost of attacking).
In the SPE, D ends the game immediately by attacking, anticipating that if it does not attack, then R will attack. This SPE assumes that $c < 1$.

d. Find the game's subgame-perfect equilibrium when $c > 1$.
If $c > 1$, D will anticipate that R will not attack if D does not attack. Hence, the SPE entails that neither player attacks.

e. Is the subgame-perfect equilibrium Pareto optimal? Does the answer depend upon whether c is bigger or smaller than 1?
The SPE is suboptimal if $c < 1$. In contrast, the SPE is Pareto optimal if $c > 1$ (both players would be better off if both refrain from attacking.

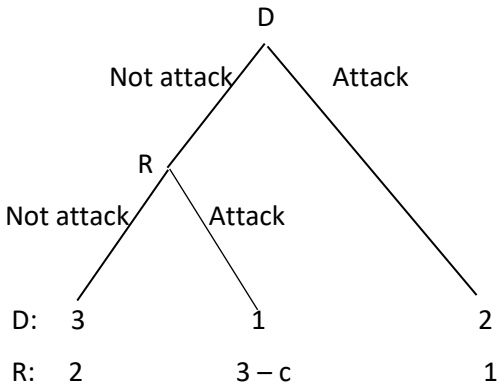


Figure 2: Question 2 – Preventive War Model.
The Figure models the Declining State (D) and the Rising State (R). D moves first, choosing to Attack or to Not Attack. If D chooses Attack, R has no choice, and D receives a payoff of 2 and R receives 1. If D chooses Not Attack, R makes a choice of Not Attack or Attack. If R chooses Not Attack, D receives 3 and R receives 2. If R chooses Attack, D receives 1, and R receives 3-c.

Question 3 (25%)
The geometric model below shows a situation in which war could be possible.

1. Explain why war is possible given the nature of this geometric model. Discuss the role of beliefs and information.

   War is possible because France does not know whether Germany is strong or weak. As portrayed in Figure 3 below, France believes that they will win a war 70% of the time if Germany is weak. France believes that they will win the war 45% if Germany is strong.

   If France demands the blue (p − CF), and Germany will accept the offer whether it is strong or weak, and there will be no war.

   If France demands the red (p−CF), and Germany is weak it will accept the offer and there will be no war. But if France demands the red (p − CF), and Germany is strong it will reject the offer and there will be war.

   This is a problem of incomplete information, whereby France does not know Germany's capabilities As portrayed in Figure 3, the costs of war are not uncertain. Only the chance of winning (p) is uncertain.

2. Using the geometric model, explain how war could be made less likely.

   If the red p and the blue p were to be the same, both parties would have the same bargaining range. Given the costs of war and similar bargaining ranges, it would be expected that the parties would find a peaceful negotiated settlement rather than go to war. In other words, France's information about Germany's capabilities is good enough that there is not much of a

distinction between a strong or a weak Germany. In addition, higher costs of war may expand the bargaining ranges, causing them to overlap and facilitate a feasible negotiated settlement.

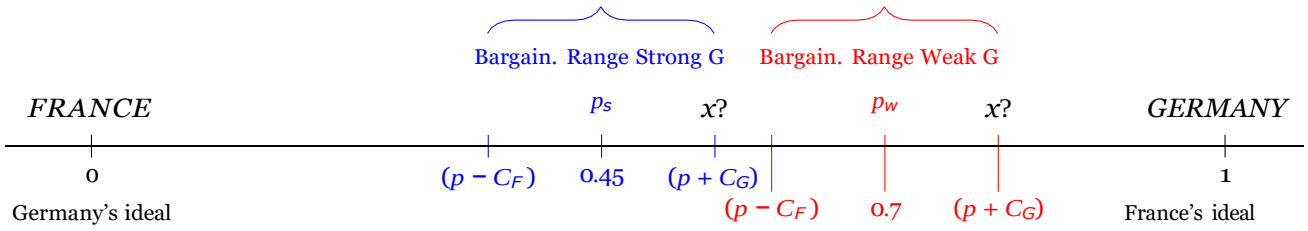


Figure 3: Question 3 – Geometric Model.
The figure depicts a line from zero to one that captures the probability of winning the conflict, the cost of war, and displays two bargaining ranges for a strong and weak type of Germany. The bargaining range for a strong Germany ranges from p_s-C_F to ps+C_G with p=0.45. The bargaining range for a weak Germany ranges from $p_w - C_F$ and $p_w + C_G$ with p_w=0.7.

Question 4 (25%)
What is a commitment problem? What are two types of commitment problems that could lead to war? Explain why these commitment problems could lead to war.

Commitment Problem:

- When actors cannot credibly commit to not use their military strength in the future, which makes it hard to find a mutually acceptable bargain.
- This commitment problem may manifest itself in two ways, preventive war (the problem of a rising power) and preemptive war (offensive advantage). In cases of power transitions, the threat of being attacked in the future may cause an actor to start a preventive war. Or when offensive military capabilities have the advantage over defense military capabilities, a first-strike advantage may lead an actor to begin a preemptive war.

Preventive War:

- A pair of countries whereby 1 is gaining strength more quickly than the other. The rising country (B) is on a trajectory to overpass the stronger power (A).
- Changing capabilities of the rising power alters the probability of victory in a war in the future.
- This rise in the bargaining position of the rising power B creates a situation in which at t1 A is much stronger than B but at t2, A and B will be even and at t3, B will be stronger than A.
- This situation creates an incentive for A to attack B at t1 when it is relatively stronger rather than waiting until t2 or t3 when B will be stronger. Such an attack would constitute a preventive war.
- Country A has a commitment problem.
- Examples: Spaniel – Israel-Iraq; Wohlford -- Germany-Russia

Preemptive war:

- Offensive advantage creates a commitment problem
- First-strike advantage outweighs the combined costs of war for both players. In such a situation, the country with an offensive advantage will attack preemptively
- Answer should make reference to the geometric model or a game tree to demonstrate the logic of a preemptive war
- Examples: Wohlford -- Japan-Russia in Russo-Japanese War (pp. 217-18); Germany – France attack via Belgium