EXAM - Solutions
STV1266

Introduction to Game-theoretic Models of War
Spring 2024

9:00
23 April 2024

Instructions: You have four hours to take the exam. Please allocate your time carefully. Answer all the questions.

Question 1 (20%)

Consider Table 1 and answer the following questions.

1. Find the game's Nash equilibrium (or equilibria) in pure strategies.

   The pure strategy Nash equilibrium is $\{(Limited, Limited), (Total, Total)\}$.

2. Does the game contain another Nash equilibrium in mixed strategies, yes or no? If yes, what is it? If no, why not?

   Yes, there is a mixed strategy Nash Equilibrium.
   Let $p = Pr(\text{Country 2 Limited})$ and $q = Pr(\text{Country 1 Limited})$.

   $$Eu_{c1}(Limited) = p \cdot 7 + (1-p) \cdot 0 = 7p$$

   $$Eu_{c1}(Total) = p \cdot 5 + (1-p) \cdot 1 = 1 + 4p$$

   Country 1 prefers to attack when:

   $$\begin{aligned} 7p &\geq 1 + 4p \\ \Rightarrow 3p &\geq 1 \\ \Rightarrow p &\geq \frac{1}{3} \end{aligned}$$

   By parallel arguments (due to the symmetry of payoffs), we find $q \geq \frac{1}{3}$. The mixed strategy Nash Equilibrium is given by Country 1 playing Limited with frequency $\frac{1}{3}$ and Total with frequency $\frac{2}{3}$ and Country 2 playing Limited with frequency $\frac{1}{3}$ and Total with frequency $\frac{2}{3}$.

3. Which outcomes are Pareto optimal?

   There is one Pareto optimal outcomes: $\{(Limited, Limited)\}$.

4. Which outcomes are suboptimal?

   There are three Pareto suboptimal outcome: $\{(Limited, Total), (Total, Limited), (Total, Total)\}$.

5. In a few sentences, discuss how this game and its results connect to coordination games and the intensity of conflict (e.g., Britain and Germany's naval warfare during World War I).

Free response. Points for discussing the best response to coordinate with move of the opponent. Points if mention limited war is welfare-maximizing individually and collectively. Also points if mention historical context, norms, and preemptive war.

		Country 2	
		Limited	Total
Country 1	Limited	7,7	0,5
	Total	5,0	1,1

Table 1: Question 1 — Intensity of Fighting Normal Form

Question 2 (30%)

Consider Figure 1 and Table 2. B is the value of waiting for both countries. Let $B = 4$ and answer the following questions.

1. Consider Figure 1. What is this game tree's subgame perfect Nash equilibrium?

   The subgame perfect Nash equilibrium is $\{(Attack, Attack)\}$.

2. Now, consider Table 2, which translates the game tree in Figure 1 to a normal form game. Find the game's Nash equilibrium (or equilibria) in pure strategies.

   The pure strategy Nash equilibrium is $\{(Attack, Attack), (Attack, Wait)\}$.

3. Does Table 2 and Figure 1 have the same equilibrium outcomes? If not, what accounts for the difference?

   The game tree is missing the $\{(Attack, Wait)\}$. This is because it is not credible for Country 2 to wait for the off-path outcome of country 1 waiting (if they play the on-path strategy of attack). In substantive terms, country 2 would never wait if they move.

Once more, consider Figure 1 and Table 2. Let $B = 9$ and answer the following questions.

1. Consider Figure 1. What is this game tree's subgame perfect Nash equilibrium?

   The subgame perfect Nash equilibrium is $\{(Wait, Wait)\}$.

2. Now, consider Table 2, which translates the game tree in Figure 1 to a normal form game. Find the game's Nash equilibrium (or equilibria) in pure strategies.

   The pure strategy Nash equilibrium is $\{(Attack, Attack), (Wait, Wait)\}$.

3. Does Table 2 and Figure 1 have the same equilibrium outcomes? If not, what accounts for the difference?

   The game tree is missing the $\{(Attack, Attack)\}$. This is because it is not credible for Country 2 to attack for the off-path outcome of country 1 waiting (if they play the on-path strategy of attack). In substantive terms, country 2 would never attack if they move.

4. How can this result connect to the commitment problem of preventive war? How does the result depend on the benefit of waiting (B)?

   Free response. Points for discussing the commitment problem of the declining power to attack now before they lose power to the rising power of country 2, who cannot commit to not attack. Can receive full points if they discuss how B shifts the commitment problem, i.e., removes it if waiting is too valuable.

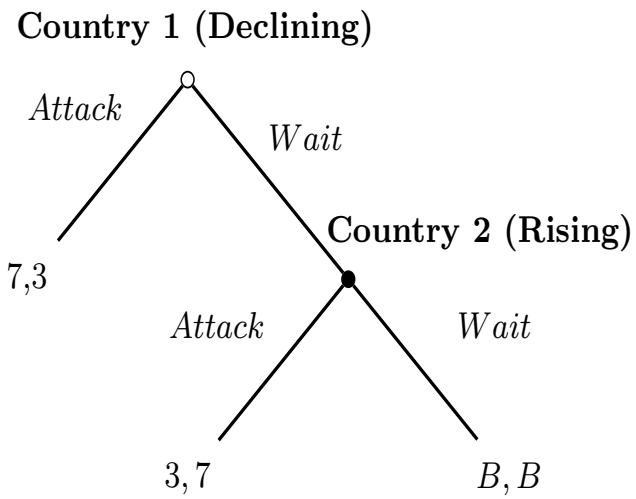


Figure 1: Question 2 Game Tree

		Country 2 (Rising)	
		Attack	Wait
Country 1 (Declining)	Attack	7,3	7,3
	Wait	3,7	B,B

Table 2: Question 2 Normal Form

Question 3 (25%)

The geometric model below shows a situation in which war could be possible.

1. Explain why war is possible given the nature of this geometric model. Discuss the role of beliefs and information.

   War is possible because France does not know whether Germany is strong or weak. As portrayed in the Figure below, France believes that they will win a war 70% of the time if Germany is weak. France believes that they will win the war 45% if Germany is strong.

   If France demands the blue $(p - C_F)$, and Germany will accept the offer whether it is strong or weak, and there will be no war.

   If France demands the red $(p - C_F)$, and Germany is weak it will accept the offer and there will be no war. But if France demands the red $(p - C_F)$, and Germany is strong it will reject the offer and there will be war.

   This is a problem of incomplete information, whereby France does not know Germany's capabilities

   As portrayed in this figure, the costs of war are not uncertain. Only the chance of winning (p) is uncertain.

2. Using the geometric model, explain how war could be made less likely.

   If the red p and the blue p were to be the same, both parties would have the same bargaining range. Given the costs of war and similar bargaining ranges, it would be expected that the parties would find a peaceful negotiated settlement rather than go to war. In other words, France's information about Germany's capabilities is good enough that there is not much of a distinction between a strong or a weak Germany. In addition, higher costs of war may expand the bargaining ranges, causing them to overlap and facilitate a feasible negotiated settlement.

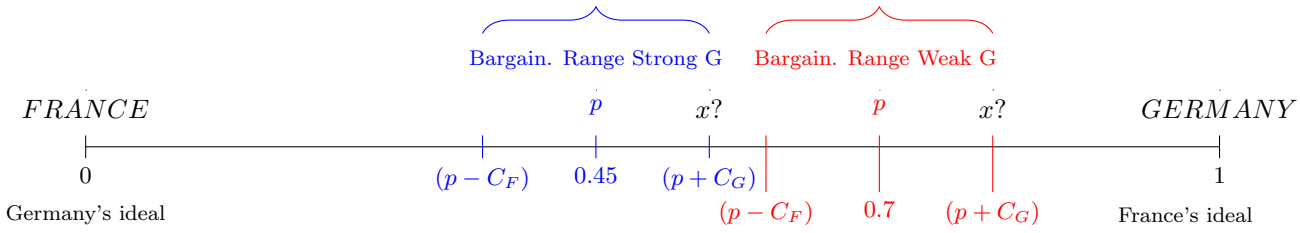


Figure 2: Question 3 – Geometric Model. The figure depicts a line from zero to one that captures the probability of winning the conflict, the cost of war, and displays two bargaining ranges for a strong and weak type of Germany.

Question 4 (25%)

War is inefficient. Yet, wars occur. Explain why wars are inefficient. Under what conditions might rational actors fight a war?

Wars are inefficient because of the costs of war. With complete information and no commitment problems, states should be able to find a mutually advantageous negotiated settlement that avoids the costs of war.

Four conditions explain how war can occur in spite of the inefficiency of war and why a bargained solution is difficult.

- Information problems:
  - Incomplete information creates uncertainty
    - Uncertainty about capabilities (probability of victory)
    - Uncertainty about resolve (costs)
  - Incomplete information will affect the perception of the bargaining range. There may be situations in which there is no bargaining range, like the Figure in Problem III of the exam. (The alternative way to answer would be to focus on the game tree and player B not knowing player A's type.
  - Misperception is a problem – signalling will help players' update their beliefs. Costly signals vs. cheap talk.
  - Problem can be compounded by deception
  - Example: Spaniel – South Osetia (Georgia vs. Russia); Wohlford – Germany-France
- Preventive War:
  - A pair of countries whereby 1 is gaining strength more quickly than the other. The rising country (B) is on a trajectory to overpass the stronger power (A).
  - Changing capabilities of the rising power alters the probability of victory in a war in the future.
  - This rise in the bargaining position of the rising power B creates a situation in which at t1 A is much stronger than B but at t2, A and B will be even and at t3, B will be stronger than A.
  - This situation creates an incentive for A to attack B at t1 when it is relatively stronger rather than waiting until t2 or t3 when B will be stronger. Such an attack would constitute a preventive war.
    - Country A has a commitment problem.
    - Example: Spaniel – Israel-Iraq; Wohlford – Germany-Russia
- Preemptive war:
  - Offensive advantage creates a commitment problem
  - First-strike advantage outweighs the combined costs of war for both players. In such a situation, the country with an offensive advantage will attack preemptively
  - Answer should make reference to the geometric model or a game tree to demonstrate the logic of a preemptive war
  - Example: Wohlford – Japan-Russia in Russo-Japanese War (pp. 217-18); Germany – France attack via Belgium
- Indivisibility:
  - The conflict is all or nothing – zero-sum
  - In a geometric model, this means that player A and player B want the entire range from 0 to 1.
  - This means that there is no bargaining range.
  - It is extremely rare
  - Example: Spaniel – US-Afghanistan over Bin Laden – Deliver Bin Laden or not.