Final Exam *(Evaluator Version)*

MAE4101 Measurement Models (Spring Term 2023)

Faculty of Educational Sciences
Centre for Educational Measurement at the University of Oslo (CEMO)

Exam date: 16 June 2023, 9:00-13:00 Oslo time, Silurveien 2, NO-0380 Oslo, Room 4D

Welcome to the MAE4101 Measurement Models exam!

Before you begin, please make sure to consider the following:

- **Read** the questions **carefully**.
- Notice which **task operators** are used (e.g., *name* versus *explain* something).
- You may **simplify subscripts** wherever appropriate (e.g., Y1 instead of Y_1).
- You may **simplify Greek letters** wherever appropriate (e.g., lambda1 instead of λ_1).
- Use **dots instead of commas** to indicate decimals.
- Keep your **explanations and descriptions brief**.
- **Partial credits** will be given.

We wish you all the best for the exam and great success in working on the tasks!

Best regards,
Kseniia, Jarl, and Ronny

Abbreviations you may want to use throughout the exam:
CFA—Confirmatory factor analysis
CFI—Comparative fit index
EFA—Exploratory factor analysis
MGCFA—Multiple groups CFA
RMSEA—Root mean square error of approximation
SEM—Structural equation model
SRMR—Standardized root mean squared residual

Name:	SUGGESTED SOLUTIONS + GRADING

Results

Task	Credits	Max. credits
Measurement models		14
Path models		18
Exploratory factor analysis		10
The MIMIC model		12
Smart recycling systems		10
STRESS across countries		16
TOTAL:		**80**

Expected time on task

- Approximate time spent by teachers: 24 questions*4 min per question = 84 min
- Estimated time for students: 84 min*2.5 = 210 min

Grading

Grade	Credits range	Percentages correct
A	80-72	100-90 %
B	71-64	89-80 %
C	63-56	79-70 %
D	55-48	69-60 %
E	47-40	59-50 %
F	39-0	49-0 %

Types of Measurement Models (14 credits)

In the following tasks, you can demonstrate your understanding of the key terms and procedures concerning measurement models.

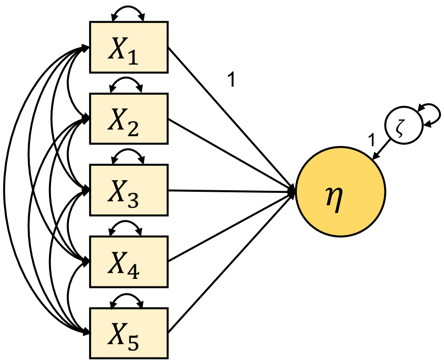
a) Define the following terms briefly.

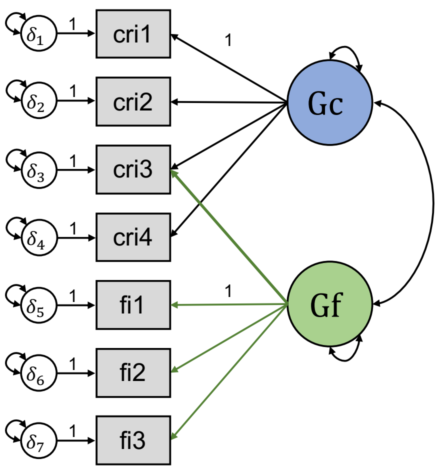
Term	Definition
Manifest variable	An observed variable that is contained in the data set. Directly observed or measured variable.
Factor loading	Pearson correlation between the factor and the indicator.
Eigenvalue	Amount of total variance in the indicators explained by each factor (with the total variance equal to the number of indicators).
Reflective measurement model	A measurement model hypothesizing that a latent variable causes the (co-)variation in the manifest (observed) variables. Examples are common factor models, such as CFA and EFA.

SCORING:

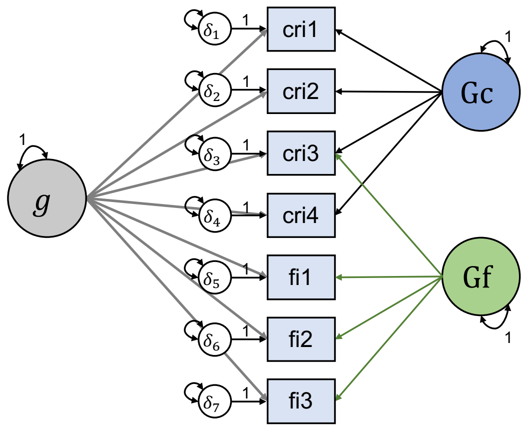
- 1 credit per correct definition
- **Sub-total: 4 credits**

b) The following figures show path diagrams of measurement models. Match the figures to the different types of measurement models.
Note: It is possible to choose multiple types of measurement models for a path diagram.

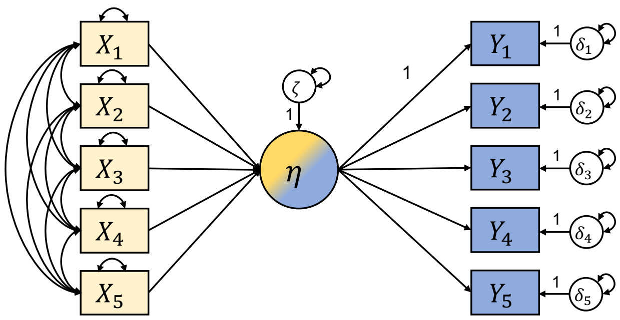
	☐ Reflective measurement model X Causal-formative measurement model ☐ Composite measurement model ☐ Not a measurement model at all



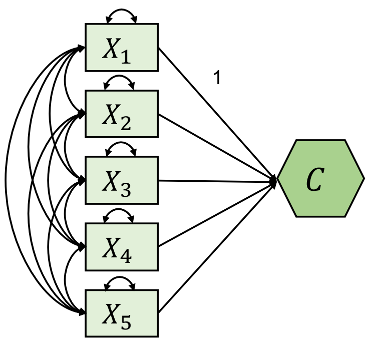
X Reflective measurement model
☐ Causal-formative measurement model
☐ Composite measurement model
☐ Not a measurement model at all



X Reflective measurement model
☐ Causal-formative measurement model
☐ Composite measurement model
☐ Not a measurement model at all



X Reflective measurement model
X Causal-formative measurement model
☐ Composite measurement model
☐ Not a measurement model at all

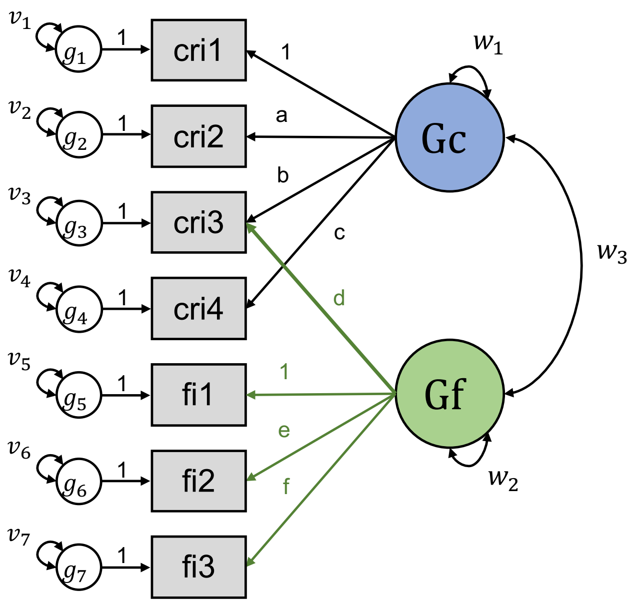


☐ Reflective measurement model
☐ Causal-formative measurement model
X Composite measurement model
☐ Not a measurement model at all

SCORING:

- 1 credit per correct selection of the measurement model(s)
- **Sub-total: 5 credits**

In the next two tasks, we focus on the following CFA measurement model that hypothesizes two correlated latent variables, each of which represent a facet of cognitive skills (Gc: crystallized intelligence, Gf: fluid intelligence):



c) Provide the **model-specifying equations** of the indicators fi1, cri1, and cri3.
Note: Ignore the mean structure.

Fill in the model equations here.

Outcome variable	Model equation
$\text{fi1} =$	$\text{Gf} + g_5$
$\text{cri1} =$	$\text{Gc} + g_1$
$\text{cri3} =$	$b\text{Gc} + d\text{Gf} + g_3$

SCORING:

- 1 credit per correct equation
- **Sub-total: 3 credits**

d) Derive the **model-implied covariance** between fi1 and cri1.
Note: You may either apply the variance-covariance rules or Wright's path tracing rules. Make sure to provide the steps you have taken to arrive at the end result.

Fill in the equations here.

Model-implied element	Result
$Cov(\text{fi1}, \text{cri1})$	$= Cov(\text{Gf} + g_5, \text{Gc} + g_1)$

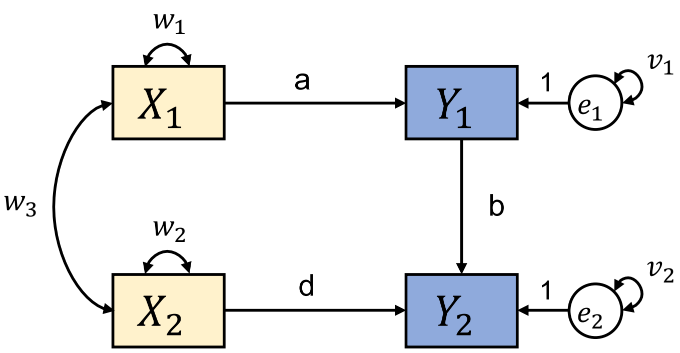
	$= Cov(\mathrm{Gf}, \mathrm{Gc}) + Cov(\mathrm{Gf}, g_1) + Cov(g_5, \mathrm{Gc}) + Cov(g_5, g_1)$ $= w_3 + 0 + 0 + 0$ $= w_3$ Using the path tracing rules, you will find that there is only one eligible pathway connecting fi1 and cri1. Start at fi1, move through the factor loading that is fixed to 1, then through the covariance between Gc and Gf (w_3), and through the factor loading of cri1. This gives $1 \cdot w_3 \cdot 1 = w_3$.

SCORING:

- 1 credit for the correct end result
- 1 credit for the correct explanation/derivation
- **Sub-total: 2 credits**

Path Models (18 credits)

The following path model contains four variables and does *not* have a mean structure.



a) Classify all variables in Model 1 as **endogenous manifest, exogenous manifest, or residual variables**.

Variable	Endogenous manifest	Exogenous manifest	Residual
X_1	☐	X	☐
X_2	☐	X	☐
Y_1	X	☐	☐
Y_2	X	☐	☐
e_1	☐	☐	X
e_2	☐	☐	X

SCORING:

- 0.5 credit per correct response
- **Sub-total: 3 credits**

b) Provide the **model-specifying equations** of the endogenous manifest variables in the path model.

Fill in the model equations here.

Outcome variable	Model equation
$Y_1 =$	$aX_1 + e_1$
$Y_2 =$	$bY_1 + dX_2 + e_2$

SCORING:

- 1 credit per correct equation
- **Sub-total: 2 credits**

c) Derive the **model-implied variance** of Y_1 and **the model-implied covariance** between X_1 and Y_1: $Var(Y_1)$ and $Cov(X_1, Y_1)$.
Note: You may apply either the variance-covariance rules or Wright's path tracing rules. Make sure to provide the steps you have taken to arrive at the end result.

Fill in the equations here.

Model-implied element	Result
$Var(Y_1)$	$= Var(aX_1 + e_1)$ $= Var(aX_1) + Var(e_1) + 2Cov(aX_1, e_1)$ $= a^2Var(X_1) + v_1 + 2aCov(X_1, e_1)$ $= a^2w_1 + v_1 + 2a \cdot 0$ $= a^2w_1 + v_1$ Using the path tracing rules, Y_1 has got only one predictor, that is X_1 with its variance w_1. Starting at Y_1 and arriving back at Y_1 has got only two possible paths that following the tracing rules: (1) Going backward through the path a, then through the variance of X_1 (w_1), going forward through a again, and arrive at Y_1. This gives $aw_1a = a^2w_1$. (2) Going backward to e_1, going through the variance of e_1 (v_1), and then forward to Y_1. This gives $1 \cdot v_1 \cdot 1 = v_1$. Both pathways add up to $a^2w_1 + v_1$.
$Cov(X_1, Y_1)$	$= Cov(X_1, aX_1 + e_1)$ $= Cov(X_1, aX_1) + Cov(X_1, e_1)$ $= aCov(X_1, X_1) + 0$ $= aVar(X_1)$ $= aw_1$ Using the path tracing rules, there is only one eligible pathway connecting X_1 and Y_1 via a and the variance of X_1. This gives aw_1.

SCORING:

- 1 credit for the correct end results
- 1 credit for the correct explanation/derivation
- **Sub-total: 4 credits**

d) Show that **the path model is (over-)identified** by providing the number of available pieces of information (p), the number of freely estimated parameters (q), and the degrees of freedom of the model (df_M).
Note: Once again, ignore the mean structure.

Elements	Numbers
# Observed pieces of information (p)	4 observed (manifest variables, X_1-X_2 and Y_1-Y_2) with 4 variances and 6 covariances ⇒ p = 10
# Parameters to be estimated (q)	To be estimated: 3 path coefficients (a, b, d); 2 residual variances (v_1 and v_2); 2 variances of the exogenous manifest variables (w_1 and w_2); 1 covariance between X_1 and X_2 (w_3) ⇒ q = 8
Degrees of freedom (df_M)	$\text{df}_M = p - q = 10 - 8 = 2 > 0$

SCORING:

- 1 credit per correct response element
- **Sub-total: 3 credits**

Another Path Model

In their recently published paper, Bråten et al. (2022) tested a path model that explained students' performance on reading comprehension ("comprehension performance") by various other variables. The path model and the standardized path coefficients are shown below.

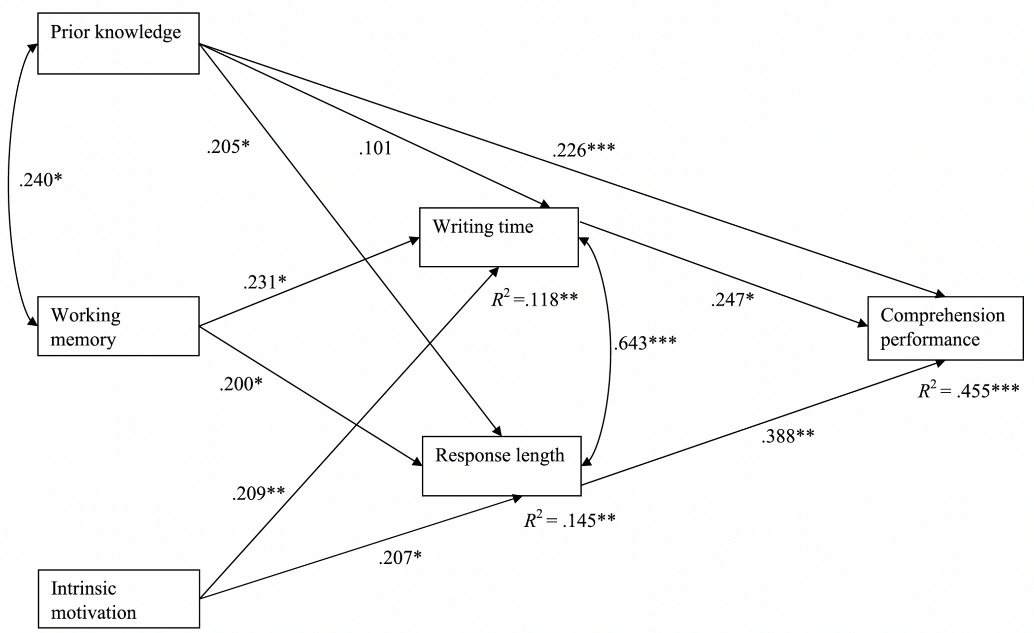


Fig. 2 The model with standardized path coefficients and explained variance of dependent variables. *$p < .05$, **$p < .01$, ***$p < .001$

Model fit: $\chi^2(4, n=116) = 0.237$, p = .993, CFI = 1.000, RMSEA = 0.000, SRMR = 0.008

Source: Bråten et al. (2022), https://doi.org/10.1007/s11145-021-10205-x, p. 701

e) Which of the following statements about this path model are **true or false**?

Statement	True	False
The variables in the model explain 45.5% of the variance in Comprehension performance.	X	☐
Writing time and Response length are exogenous manifest variables in the model.	☐	X
The chi-square test indicates that the model does not fit the data well.	☐	X
Within the model, the relation between Prior knowledge and Comprehension performance is positive and statistically significantly different from zero.	X	☐

NOTES:

- **Statement 1:** TRUE. The R-squared value of .455 indicates the amount of variation explained in Comprehension performance.
- **Statement 2:** FALSE. These two variables have predictors and are thus endogenous manifest variables.
- **Statement 3:** FALSE. This test indicates that there is no difference between the observed and model-implied covariance matrix. Hence, the model fits the data well.
- **Statement 4:** TRUE. The path coefficient is .226**.

f) The authors implemented this path model in the R package lavaan. Please find the model specification code below.

Note: They labelled the model "Path.Model" and used the following abbreviations for the variables:

- *Comprehension performance (CP)*
- *Writing time (WT)*
- *Response length (RL)*
- *Prior knowledge (PK)*
- *Working memory (WM)*
- *Intrinsic motivation (IM)*

Lavaan code for the model specification:

```
Path.Model <-  '
               CP ~ WT + RL
               WT ~ PK + WM + IM
               RL ~ PK + WM + IM

               PK ~~ WM
               '
```

This specification code contains **two errors**. Provide the correct specification code.

Corrected lavaan code for the model specification:

```
Path.Model <-  '
               CP ~ WT + RL + PK
               WT ~ PK + WM + IM
               RL ~ PK + WM + IM

               PK ~~ WM
               WT ~~ RL
               '
```

SCORING:

- 1 credit for adding PK to the CP regression command
- 1 credit for adding the residual covariance between WT and RL
- **Sub-total: 2 credits**

Exploratory Factor Analysis (10 credits)

In the following tasks, you can demonstrate your understanding of exploratory factor analysis.

Steps in an EFA

a) Name **four key steps** in an EFA.

Four steps
Four of the following steps: Testing of prerequisites, factor enumeration, factor extraction, factor rotation, factor interpretation, and factor scores

SCORING:

- 0.5 credits per correct step
- **Sub-total: 2 credits**

Prerequisites for an EFA

b) Several tests of the prerequisites for an EFA exist. Describe what **a significant Bartlett test of sphericity** indicates.

What does a significant Bartlett's Test of Sphericity indicate?
A significant test indicates that the correlation matrix is not the identity matrix. Hence, there is "sufficient" correlation in the data.

SCORING:

- 1 credit for the correct description
- Note: Both formulations (statistical and conceptual) can be considered for full credits.
- **Sub-total: 1 credit**

Oblique vs. orthogonal

c) What is the difference between the oblique and orthogonal rotation? Explain briefly.

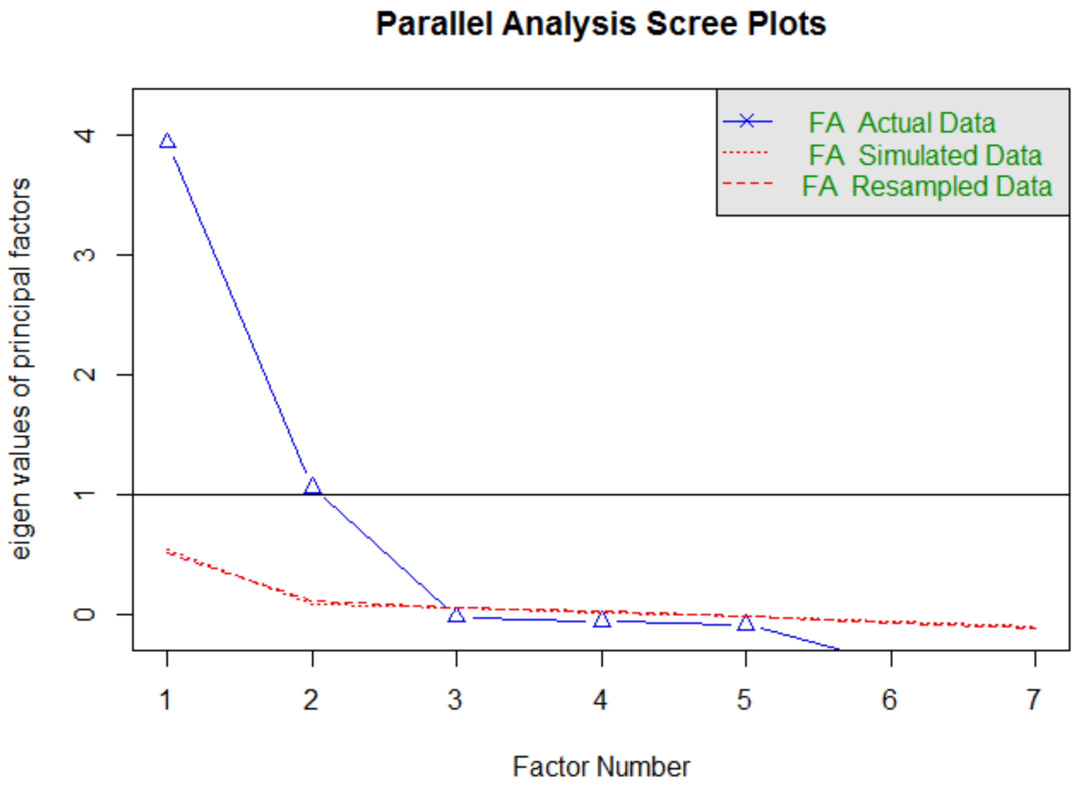
Difference oblique vs. orthogonal
Assumption of correlated (oblique) vs. independent (orthogonal) factors

SCORING:

- 1 credit for the correct description
- **Sub-total: 1 credit**

Number of factors

d) Using the information about the results of an EFA below, decide on **how many factors should be extracted** according to the different criteria.




```
EMPIRICAL KAISER CRITERION

Kind of correlations analyzed: Pearson

   Nfactors     Eigenvalue     Reference Values
          1          4.212                1.233
          2          1.760                1.185
          3          0.403                1.130
          4          0.309                1.064
          5          0.134                1.000
          6          0.096                1.000
          7          0.085                1.000
```

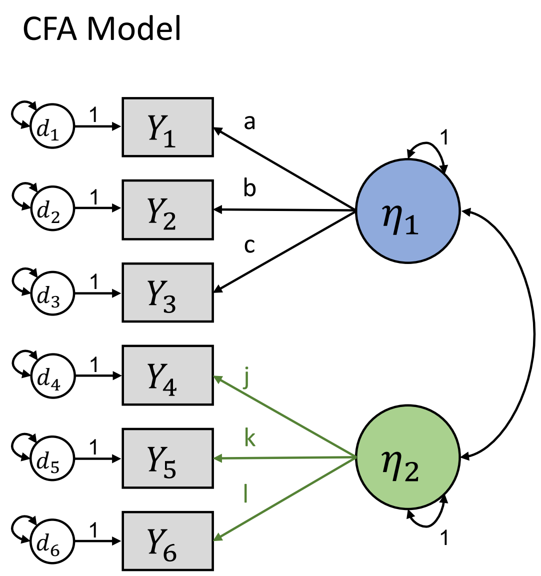
Criterion	Number of factors
Scree plot	2
Parallel analysis	2
Kaiser-Guttman Criterion	2
Empirical Kaiser Criterion	2

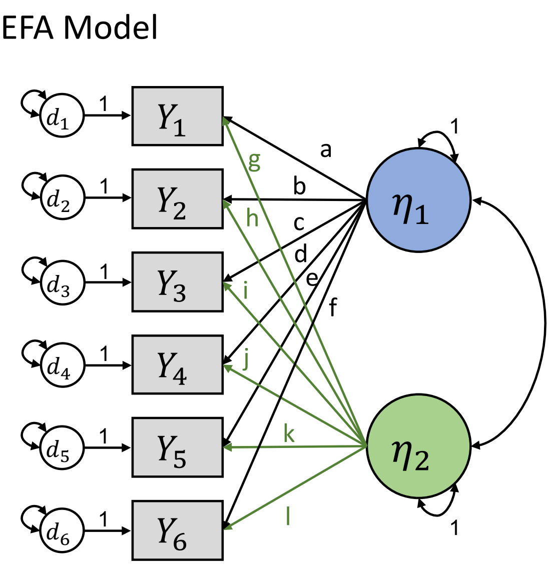
SCORING:

- 1 credit per correct number of factors
- **Sub-total: 4 credits**

CFA vs. EFA

e) The following path diagrams show models of CFA and EFA.





Note: In the EFA model, the factor loadings of η_1 (eta1) are labelled a-f (in black), and the factor loadings of η_2 (eta2) are labelled g-l (in green). There is no mean structure in these models.

Provide the **model-specifying equations** for the indicator Y_5 in the CFA and the EFA model.

Fill in the model equations here.

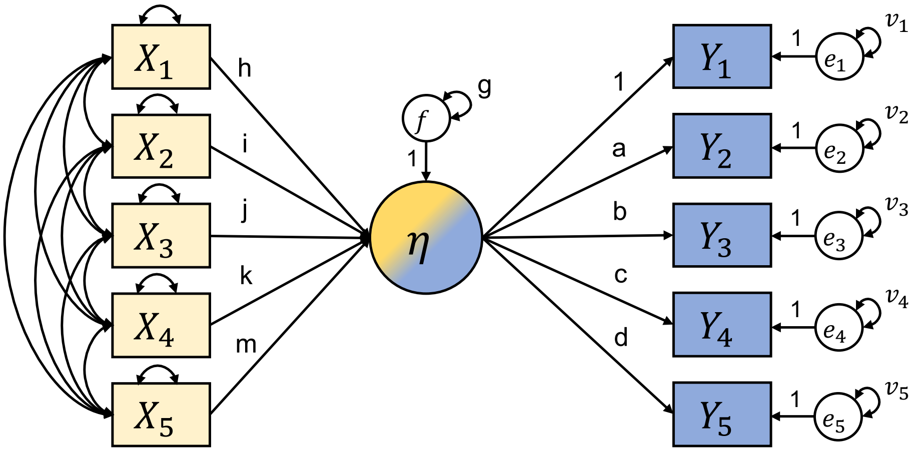
Model	Model equation
CFA	$Y_5 = k\eta_2 + d_5$
EFA	$Y_5 = e\eta_1 + k\eta_2 + d_5$

SCORING:

- 1 credit per correct equation
- **Sub-total: 2 credits**

The MIMIC Model (12 credits)

The following Multiple-Indicators-Multiple-Causes (MIMIC) model contains indicators Y_1-Y_5 of a latent variable η (eta) and predictors X_1-X_5. This model does *not* have a mean structure.



a) Provide the **model-specifying equations for all outcome variables** in this MIMIC model.

Fill in the model equations here.

Outcome variable	Model equation
$Y_1 =$	$\eta + e_1$
$Y_2 =$	$a\eta + e_2$
$Y_3 =$	$b\eta + e_3$
$Y_4 =$	$c\eta + e_4$
$Y_5 =$	$d\eta + e_5$
$\eta =$	$hX_1 + iX_2 + jX_3 + kX_4 + mX_5 + f$

SCORING:

- 1 credit per correct equation
- **Sub-total: 6 credits**

b) Which of the following statements about this MIMIC model are **true or false**?

Statement	True	False
The residuals of the indicators Y_1-Y_5 are correlated.	☐	X
The residuals of the indicators Y_1-Y_5 are assumed to follow a multivariate normal distribution with a mean vector of zeros and a covariance matrix that contains the variances v_1-v_5 on the diagonal and zeros off the diagonal.	X	☐

The residual of η (eta) is independent of the residuals of the indicators Y_1-Y_5.	X	☐
The variance of the residual of η (eta) is fixed to 1.	☐	X

NOTES:

- **Statement 1**: FALSE. In the path diagram of the MIMIC model, e_1-e_5 do not have any covariance (correlation) in the MIMIC model.
- **Statement 2:** TRUE. This is the standard assumption on residuals in a reflective measurement model.
- **Statement 3:** TRUE. There is no connection between the residuals f and e_1-e_5.
- **Statement 4**: FALSE. The path diagram shows that its variance g is freely estimated.

SCORING:

- 1 credit per correct response
- **Sub-total: 4 credits**

c) Describe how you could test the indirect effect of X_2 on Y_2 via η (i.e., $X_2 \rightarrow \eta \rightarrow Y_2$).

Description
▪ Estimate the indirect effect ia ▪ Test ia against zero, using a Wald test

SCORING:

- 1 credit for specifying the indirect effect as i*a
- 1 credit for mentioning that the indirect effect is tested against zero (i.e., significance test against zero)
- **Sub-total: 2 credits**

Smart Recycling Systems (10 credits)

Liu and Hsu (2022) studied the motivating factors behind the public's use of smart recycling systems. They developed a SEM to test their hypotheses about the motivating factors. The model with the fully standardized parameters is shown below.

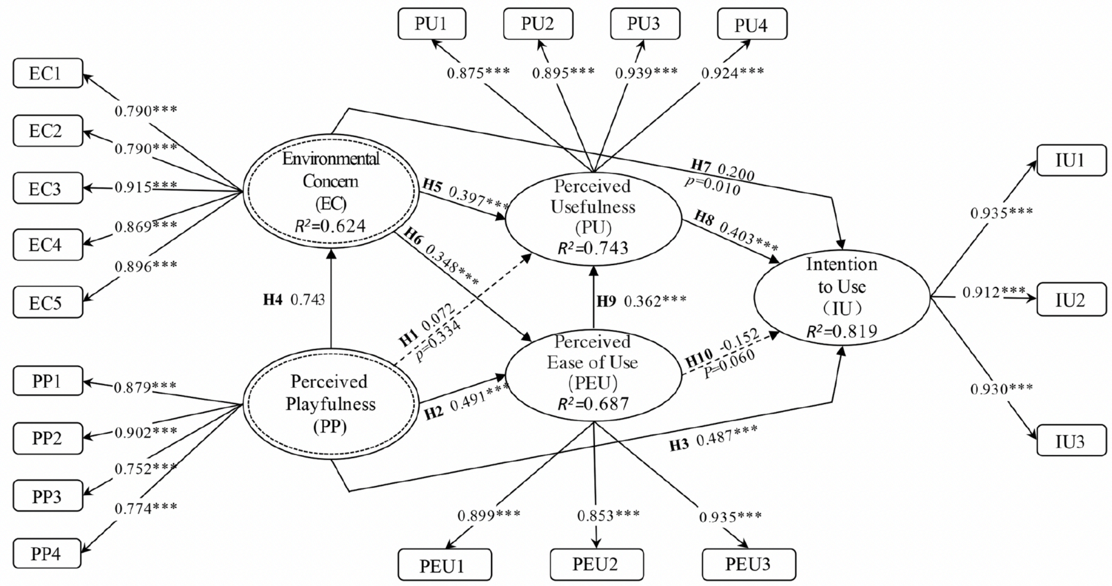


Fig. 4 Research structure pattern diagram. This picture shows the important relationships between the variables in the structural model. This figure is not covered by the Creative Commons Attribution 4.0 International License. Reproduced with permission of Liyuan Liu; copyright © Liyuan Liu, all rights reserved.

Source: Liu & Hsu (2022), https://doi.org/10.1057/s41599-022-01347-6, p. 8

a) Provide a **lavaan code** that specifies this model.

Note: The dashed lines indicate insignificant paths. Please include them in the model specification. The model is labelled "SRS.Model". You may use the abbreviations shown in the figure. You can decide on how to set the scale of the latent variables. Do not fix the model parameters to the values in the figure!

Lavaan code for model specification:

```
SRS.Model <- '
              # Measurement models
              IU =~ IU1 + IU2 + IU3
              PU =~ PU1 + PU2 + PU3 + PU4
              PEU =~ PEU1 + PEU2 + PEU3
              EC =~ EC1 + EC2 + EC3 + EC4 + EC5
              PP =~ PP1 + PP2 + PP3 + PP4

              # Structural model
              IU ~ EC + PU + PEU + PP
              PU ~ EC + PP + PEU
              PEU ~ EC + PP
              EC ~ PP

              '
```

SCORING:

- 1 credit per correct measurement model (5 credits)
- 1 credit per correct structural model part (4 credits)
- 0.5 credits per code line will be deducted for additional yet incorrect lines of code.
- **Sub-total: 9 credits**

b) Prior to estimating the full structural equation model, the authors performed a CFA with all latent variables. The five latent variables were correlated, and the unstandardized factor loadings of the first indicators were fixed to 1 to set the scales. The fully standardized factor loadings (labelled as "**Std.**") are shown below.

Table 3 Analysis of the measurement model results.

Construct	Item	SE	*p*-value	Std.	CR	AVE
EC	EC1			0.790	0.930	0.729
	EC2	0.075	0.000	0.790		
	EC3	0.065	0.000	0.915		
	EC4	0.071	0.000	0.869		
	EC5	0.070	0.000	0.896		
PU	PU1			0.875	0.950	0.826
	PU2	0.061	0.000	0.895		
	PU3	0.047	0.000	0.939		
	PU4	0.055	0.000	0.924		
PP	PP1			0.879	0.898	0.688
	PP2	0.052	0.000	0.902		
	PP3	0.065	0.000	0.752		
	PP4	0.065	0.000	0.774		
PEU	PEU1			0.899	0.910	0.771
	PEU2	0.054	0.000	0.881		
	PEU3	0.058	0.000	0.853		
IU	IU1			0.935	0.947	0.857
	IU2	0.041	0.000	0.912		
	IU3	0.037	0.000	0.930		

EC environmental concern, *PU* perceived usefulness, *PP* perceived playfulness, *PEU* perceived ease of use, *IU* intention to use, *SE* standard error, *Std.* Standardized factor loadings, *CR* composite reliability, *AVE* average variance extracted.

Source: Liu & Hsu (2022), https://doi.org/10.1057/s41599-022-01347-6, p. 6

Focusing on the measurement model of the intention to use (IU) smart recycling systems, how would you calculate the **model-implied correlation between the indicators IU1 and IU2**?

Note: You only need to provide the calculation, not the end result. Please bear in mind that the table shows the fully standardized CFA solution. This implies that the manifest and latent variables in this model have a variance of 1.

Calculation of the model-implied correlation:
The model-implied correlation is the product of the standardized factor loadings. $r(IU1, IU2) = 0.935 \times 0.912$

SCORING:

- 1 credit for the correct calculation
- **Sub-total: 1 credit**

STRESS across Countries (16 credits)

A team of social scientists conducted a study to adapt a stress scale that was originally developed in the USA to a different cultural context, specifically Singapore. The so-called **STRESS scale** consists of five items:

- Workload
- EmotionalPressure
- RoleConflict
- OrganizationalChange
- Uncertainty

The researchers aimed to establish the invariance of the STRESS measure across the two countries (i.e., USA and Singapore). The initial analysis indicated that a **reflective measurement model with a single latent variable fits reasonably well** in both the USA and Singapore separately. Now, the researchers want to test for the invariance of the STRESS measure using a multiple groups CFA.

a) Describe the following three levels of measurement invariance by specifying the **type of equality constraints** and which **conclusions can be drawn** if this invariance level is achieved.

Level	Equality constraints	Conclusions
Configural	- No constraints AND/OR all parameters are freely estimated	- Same number of factors between countries, same numbers and patterns of salient loadings
Metric	- Factor loadings are constrained to be equal between the USA and Singapore	- The relationship to the construct of STRESS is comparable between the USA and Singapore
Scalar	- Factor loadings are constrained to be equal between the USA and Singapore - Indicator intercepts are constrained to be equal between the USA and Singapore	- Countries are comparable on their means on the STRESS scale

SCORING:

- 1 credit for the correctly specifying the constraints in each step [0.5 credit if the constraints of the previous steps are not transferred to the next step)
- 1 credit for the correct conclusions (for strict invariance, either of the mentioned conclusions gains 1 credit)
- **Sub-total: 6 credits**

b) Assuming that the researchers tested and achieved metric invariance (the model `m_metric`), provide the lavaan syntax for (i) specifying the single-factor model of the STRESS scale; (ii) estimating the model with the mean structure assuming scalar invariance; and (iii) comparing the scalar model (`m_scalar`) to the metric model (`m_metric`). Notice that the grouping variable is labelled `COUNTRY`.

Lavaan code for model specification:

```
m <-  '
      STRESS =~ Workload + EmotionalPressure +
      RoleConflict + OrganizationalChange +
      Uncertainty
      '
```

Lavaan code for model estimation:

```
m_scalar <- cfa( m,
                 data = data,
                 estimator = ML,
                 group = "COUNTRY",
                 meanstructure = TRUE,
                 group.equal = c("loadings", "intercepts")
                 )
```

Lavaan code for model comparison:

```
anova(m_metric, m_scalar)
```

SCORING:

- 1 credit for correct model formulation
- 1 credit for specifying meanstructure = TRUE
- 1 credit for specifying group = "COUNTRY"
- 1 credit for specifying "loadings"
- 1 credit for specifying "intercepts"
- 1 credit for correct model comparison code
- **Sub-total: 6 credits**

c) Upon comparing the metric model (`m_metric`) and the scalar model (`m_scalar`), you obtained the following R output:

	AIC	BIC	Chisq	Chisq_diff	Pr(>Chisq)
m_metric	18869	19183	303.468	-	-
m_scalar	18882	19185	340.125	36.657	0.0002538

(a) Does scalar invariance hold between countries?

☐ Yes
X No

(b) Provide evidence for your conclusion on whether scalar invariance holds.

- The Chi-square difference test suggests that there are significant differences in means between two countries (H_0: there are no differences between groups), as indicated by the low *p*-value (0.0002538) **OR**

- The Chi-square difference test suggests that the fit of the scalar model significantly deteriorated when comparing it to the metric model - The metric model (m_metric) has slightly better fit compared to the scalar model (m_scalar) based on lower AIC and BIC values.
(c) Given your previous responses, what is the main implication for comparing the responses on the STRESS scale of the USA and Singapore samples?
- The means of the groups cannot be compared as there are systematic group differences in indicator intercepts.

SCORING:

- 1 credit for correctly responding on (a)
- 1 credit for correctly interpreting the chi-square difference test results in (b)
- 1 credit for correctly interpreting AIC and BIC values in (b)
- 1 credit for specifying that the groups cannot be compared on their means (c)
- **Sub-total: 4 credits**