Final Exam (Grading Guide)

MAE4101 Measurement Models (Spring Term 2026)

Faculty of Educational Sciences
Centre for Educational Measurement at the University of Oslo (CEMO)

Exam date: 08 June 2026, 09:00-13:00, Silurveien 2, NO-0380 Oslo, Sal 4D

Welcome to the MAE4101 Measurement Models exam!

Before you begin, please make sure to consider the following:

- **Read** the questions **carefully**.
- Notice which **task operators** are used (e.g., *name* versus *explain* something).
- You may **simplify subscripts** wherever appropriate (e.g., Y1 instead of Y_1).
- You may **simplify Greek letters** wherever appropriate (e.g., lambda1 instead of λ_1).
- Use **dots instead of commas** to indicate decimals.
- Keep your **explanations and descriptions brief**.
- **Partial credits** will be given.
- For simplicity, χ^2 can be written as **chi-square**.

Abbreviations you can use without explanation throughout the exam:
CFA—Confirmatory factor analysis
EFA—Exploratory factor analysis
MGCFA—Multi-group CFA
SEM—Structural equation model
df—Degrees of freedom

We wish you all the best for the exam and great success in working on the tasks!

Best regards,
Ronny

Name:	SUGGESTED SOLUTIONS + GRADING GUIDE

Results

Task	Credits	Max. credits
Concepts, Terms, and Procedures		8
Behavioral Science for Climate Change		20
Teacher Knowledge Test		8
Assessing Academic Self-Concept		16
Intervening on Children's Mindset and Stereotypes		6
A MIMIC Model		12
TOTAL:		**70**

Expected time on task

- Approximate max. time spent by teachers: 60 minutes
- Estimated max. time for students: 60*2.5 = 150 minutes

Grading

Grade	Credits range	Percentages correct
A	70-63	100-90%
B	62-56	89-80%
C	55-49	79-70%
D	48-42	69-60%
E	41-35	59-50%
F	34-0	49-0%

Concepts, Terms, and Procedures (8 credits)

In the following tasks, you can demonstrate your understanding of **key concepts, terms, and procedures** concerning structural equation models.

a) Define the following terms briefly.

Term	Definition
Reflective measurement model	In a reflective measurement model, the latent (unobserved) construct is viewed as the cause of its manifest (observed) indicators.
Indirect effect	A derived parameter that represents the indirect relation between two variables via other variables. Typically, the path coefficients connecting the variables are multiplied to get an estimate of an indirect effect.
Partial mediation	Partial mediation occurs when an independent variable (*X*) influences a dependent variable (*Y*) both directly and indirectly through a mediator (*M*).
Residual variance	Residual variance (often termed error variance) is the portion of an observed variable's total variance that is not explained by the latent factors or predictors in the model.
Factor correlation	Factor correlation is a measure of the linear association between two latent variables that are defined reflectively.

SCORING:

- 1 credit per correct definition
- *Note: Deviations from the suggested deviations may occur.*
- **Sub-total: 5 credits**

b) The **Comparative Fit Index (CFI)** of a structural equation model was 0.985. What does this result mean for the fit of the model?

Interpretation
According to the overall cut-off values for the CFI (0.95), this result suggests a good model fit.

SCORING:

- 1 credit for the correct interpretation of this finding
- **Sub-total: 1 credit**

c) Describe which elements and parameters in a factor model are constrained to be equal across groups in a **metric invariance model**.

Description
Metric invariance: configural invariance (i.e., same number of factors and same patterns of links between the manifest variables and factors) plus the equality of factor loadings

SCORING:

- 1 credit for the correct configural constraint
- 1 credit for the correct mentioning of the equality of factor loadings
- Note: Should the mentioning of the specific elements that are the same across groups in the configural model be missing but "configural invariance" is mentioned as a prerequisite, only 0.5 credit is deducted.
- **Sub-total: 2 credits**

Behavioral Science for Climate Change (20 credits)

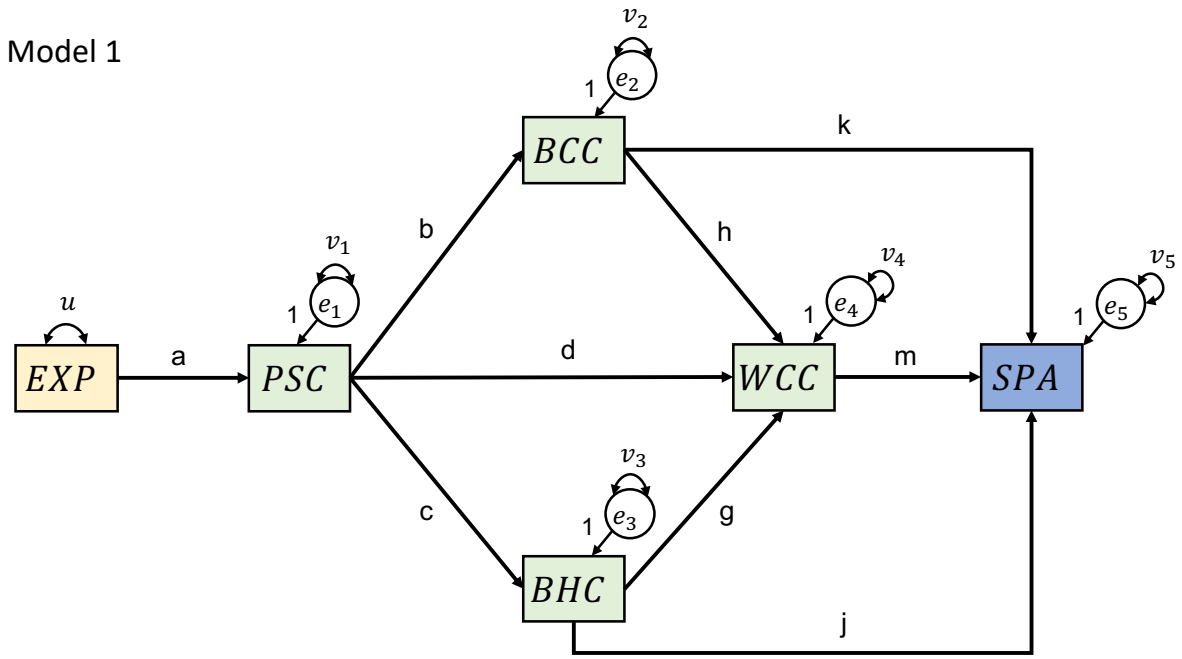
In a study of 12795 adults, Rode et al. (2025) tested the so-called "Gateway Belief Model" in the context of climate change. This model includes following variables:

- **EXP**: Treatment **exp**osure
- **PSC**: **P**erceived **S**cientific **C**onsensus
- **BCC**: **B**elief in **C**limate **C**hange
- **BHC**: **B**elief in **H**uman **C**ausation
- **WCC**: **W**orry about **C**limate **C**hange
- **SPA**: **S**upport for **P**ublic **A**ction

This theory hypothesizes the following relations among these variables (see **Model 1**).

Note: This model does not have a mean structure.

Model 1



a) Classify the following variables in Model 1 as **endogenous manifest, exogenous manifest,** or **residual variables**. Check the boxes below.

Variable	Endogenous manifest	Exogenous manifest	Residual
EXP	☐	X	☐
PSC	X	☐	☐
BCC	X	☐	☐
BHC	X	☐	☐
WCC	X	☐	☐

SPA	X	☐	☐
e_1	☐	☐	X
e_4	☐	☐	X

SCORING:

- 0.5 credit per correct response
- **Sub-total: 4 credits**

b) Provide the **model-specifying equations** of Model 1.

Fill in the model equations here.

Outcome variable	Model equation
$PSC =$	$a \cdot EXP + e_1$
$BCC =$	$b \cdot PSC + e_2$
$BHC =$	$c \cdot PSC + e_3$
$WCC =$	$d \cdot PSC + h \cdot BCC + g \cdot BHC + e_4$
$SPA =$	$m \cdot WCC + k \cdot BCC + j \cdot BHC + e_5$

SCORING:

- 1 credit per correct equation
- **Sub-total: 5 credits**

c) Derive the **model-implied covariance** between EXP and PSC: $Cov(EXP, PSC)$.

Note: You may apply either the variance-covariance rules or Wright's path tracing rules. Make sure to provide the steps you have taken to arrive at the result.

Model-implied covariance	Result
$Cov(EXP, PSC)$	$= Cov(EXP, a \cdot EXP + e_1)$ $= Cov(EXP, a \cdot EXP) + Cov(EXP, e_1)$ $= a \cdot Cov(EXP, EXP) + 0$ $= a \cdot Var(EXP)$

	$= au$ *Alternative:* Using the path tracing rules, there is one eligible pathway connecting EXP and PSC via a and the variance of EXP, named u. This gives au.

SCORING:

- 1 credit for the correct end results
- 1 credit for the correct explanation/derivation steps
- **Sub-total: 2 credits**

d) Show that **Model 1 is (over-)identified** by providing the number of available pieces of information (*p*), the number of freely estimated parameters (*q*), and the degrees of freedom of the model (*df*).

Note: Once again, ignore the mean structure.

Elements	Numbers
Number of observed pieces of information (p)	6 observed (manifest) variables with 6 variances and 15 covariances ⇒ p = 21
Number of freely estimated parameters in the model (q)	To be estimated: 9 path coefficients (a, b, c, d, g, h, k, j, m); 5 residual variances (v_1-v_5); one variance of the exogenous manifest variable EXP (u) ⇒ q = 15
Degrees of freedom of the model (*df*)	$\mathrm{df}_M = p - q = 21 - 15 = 6 > 0$

SCORING:

- 1 credit per correct response element p and q
- 1 credit for the correct df
- **Sub-total: 3 credits**

e) Provide the **lavaan code** for specifying Model 1.

*Note: While you may skip the residual variances in the code, do not forget to **specify the variances of and/or covariances among the exogenous variables** in the model. There is no need to label the path coefficients, and you do not have to include any indirect effect.*

Lavaan code for the model specification

```
Model1 <-  '
           PSC ~ EXP
           BCC ~ PSC
           BHC ~ PSC
```

```
              WCC ~ PSC + BCC + BHC
              SPA ~ WCC + BCC + BHC
              EXP ~~ EXP

              '
```

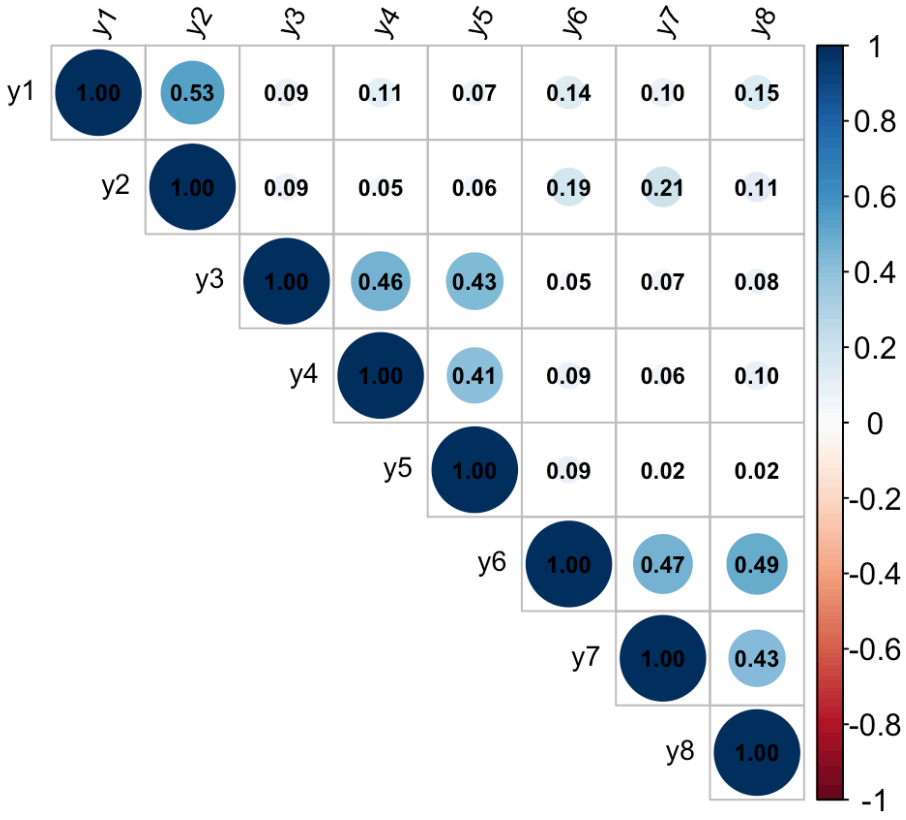
SCORING:

- 1 credit for each line specifying the structural relations
- **Sub-total: 6 credits**

Teacher Knowledge Test (8 credits)

In a study of 500 teachers, researchers administered a knowledge test. This test contained 8 questions (y1-y8), which captured three aspects of knowledge: pedagogical, content, and pedagogical content knowledge. Given that this test was just developed by the researchers, they explored its factor structure using an EFA but anticipated three correlated factors.

a) As a first step, the researchers estimated the **correlations among the eight items**. The resulting correlation matrix (with Pearson correlation coefficients) is shown below.



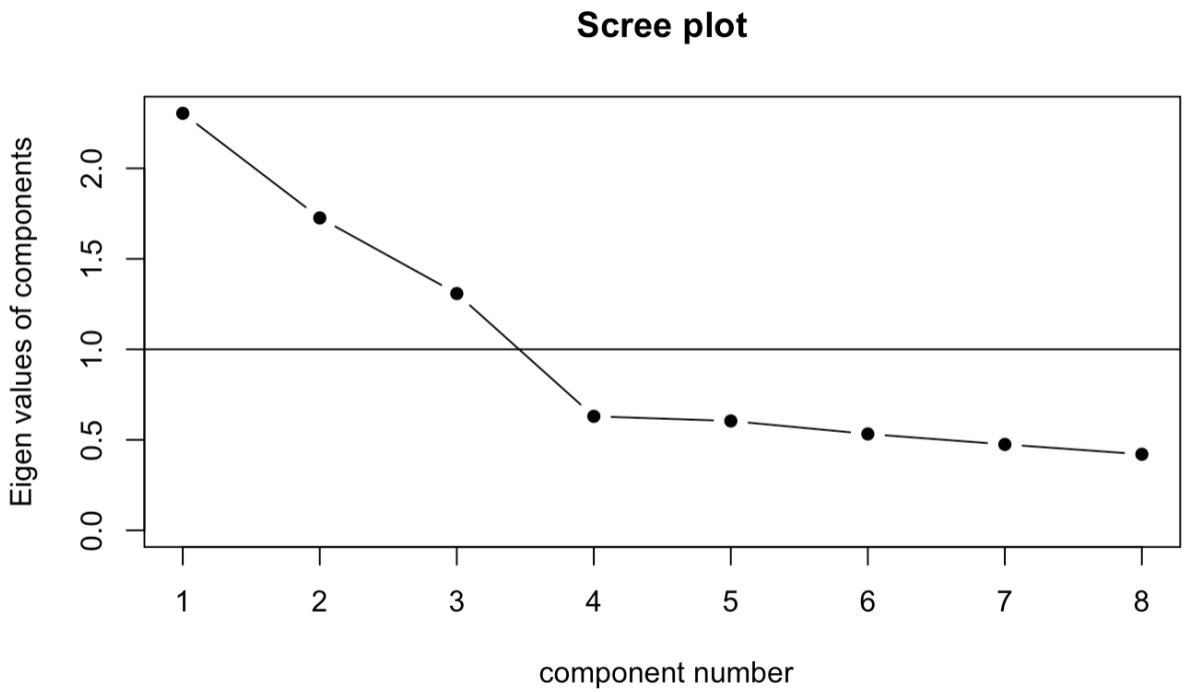
Explain why this correlation matrix shows some evidence on a factor structure with likely three factors.

Your explanation
There are three clusters/groups of items with substantial correlations: y1 and y2; y3-y5; and y7 and y8. Correlations among items in these groups are moderate, and correlations among items across groups are small and close to zero.

SCORING:

- 1 credit for the correct explanation
- **Sub-total: 1 credit**

b) An exploratory factor analysis of these data resulted in the **scree plot** below. Given this scree plot, explain why three factors may underlie the data.

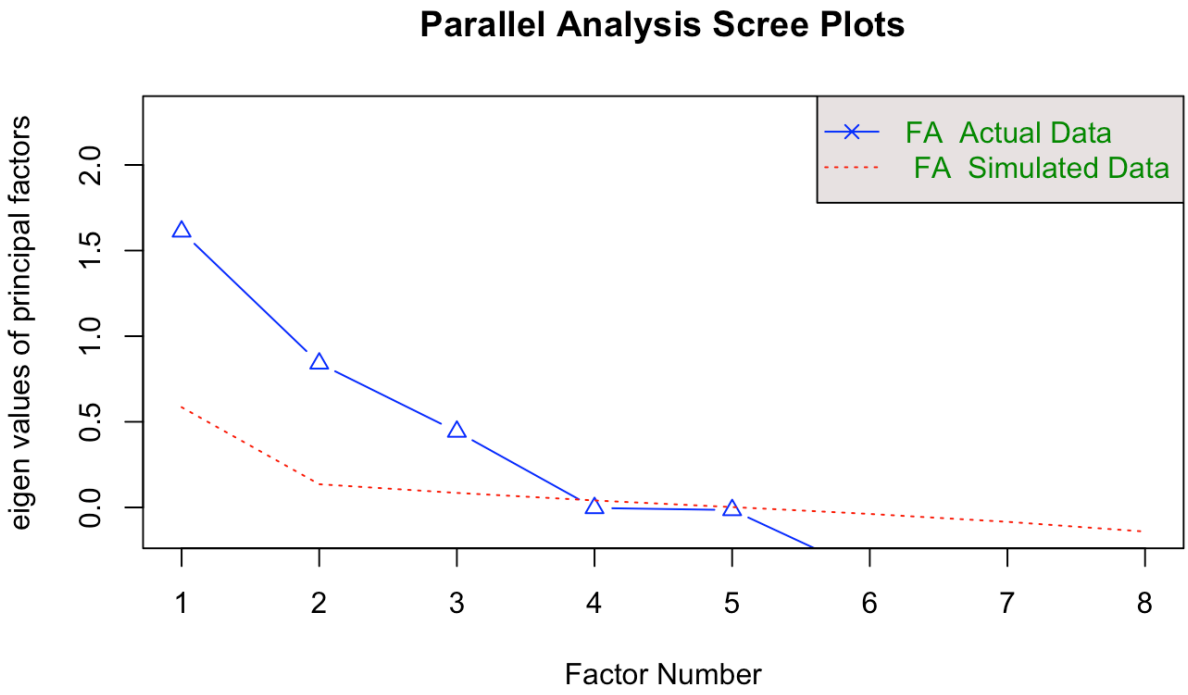


Your explanation
At least one of the two explanations: ▪ The screeplot shows a knee, until the eigenvalue curve flattens out from 4 factors onwards. ▪ There are 3 eigenvalues above 1 (Kaiser criterion).

SCORING:

- 1 credit for the correct explanation
- **Sub-total: 1 credit**

c) To decide on the number of factors in an EFA, researchers could also perform a **parallel analysis**. The following figures shows the outcomes of a parallel analysis for the data set.



Describe briefly how you could infer on the existence of 3 factors from this graphical representation of the parallel analysis.

Your description
Three eigenvalues of the data (blue line, generated from a factor analysis) are larger than the eigenvalues of the simulated data (red line).

SCORING:

- 1 credit for the correct explanation
- **Sub-total: 1 credit**

d) Using the Oblimin rotation, the following **factor loading pattern** and **factor correlations** were generated.

```
## Standardized loadings (pattern matrix) based upon correlation matrix
##      ML2   ML3   ML1   h2    u2 com
## y1  0.05  0.07  0.51 0.29 0.706 1.1
## y2 -0.01 -0.01  1.00 1.00 0.005 1.0
## y3 -0.02  0.70  0.02 0.49 0.514 1.0
## y4  0.03  0.67 -0.02 0.45 0.551 1.0
## y5 -0.02  0.62  0.00 0.38 0.618 1.0
## y6  0.72  0.00  0.01 0.53 0.469 1.0
## y7  0.62 -0.02  0.06 0.41 0.591 1.0
## y8  0.69  0.01 -0.05 0.47 0.534 1.0
```

```
##  With factor correlations of
##      ML2  ML3  ML1
## ML2 1.00 0.14 0.24
## ML3 0.14 1.00 0.11
## ML1 0.24 0.11 1.00
```

Indicate which items (y1-y8) belong to Factor 1 (labelled as ML1), Factor 2 (labelled as ML2), and factor 3 (ML3). Please check the respective boxes.

Item	Factor 1 (ML1)	Factor 2 (ML2)	Factor 3 (ML3)
y1	X	☐	☐
y2	X	☐	☐
y3	☐	☐	X
y4	☐	☐	X
y5	☐	☐	X
y6	☐	X	☐
y7	☐	X	☐
y8	☐	X	☐

SCORING:

- 0.25 credit for the correct response
- **Sub-total: 2 credits**

e) Using the results from the EFA, the researchers wanted to evaluate the **factor structure via CFA** for a new sample of teachers. Provide the lavaan code they could specify based on the findings from the EFA.
Note: You do not need to provide the lavaan code for specifying an EFA.

Lavaan code for the model specification

```
CFA <-  '
        ML1 =~ y1+y2
        ML2 =~ y3+y4+y5
        ML3 =~ y6+y7+y8
        ML1 ~~ ML2+ML3
        ML2 ~~ ML3
        '
```

SCORING:

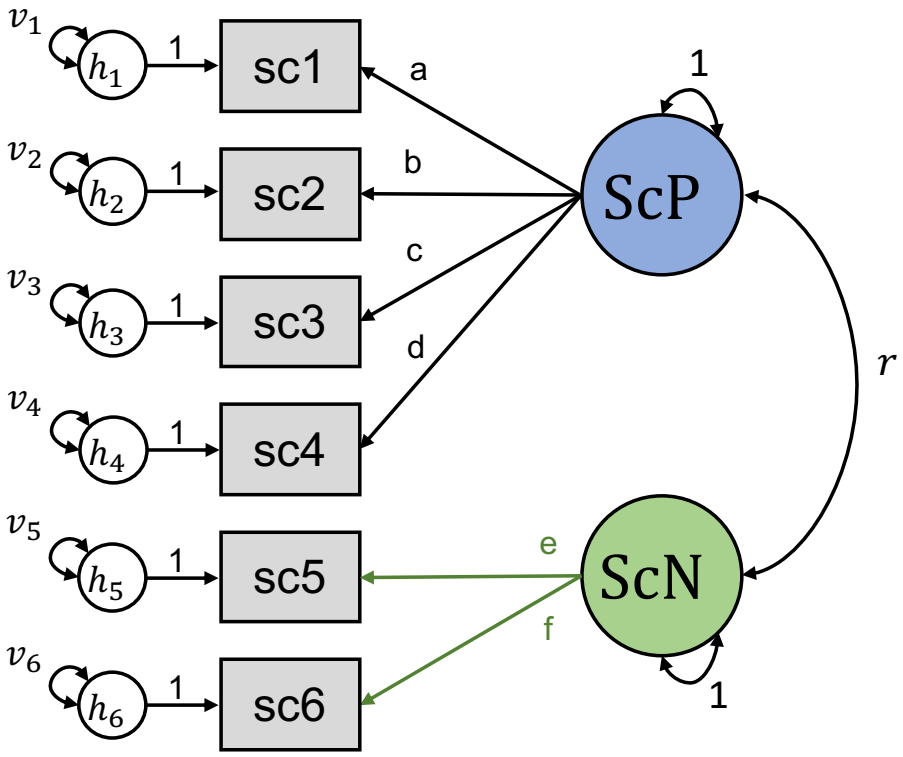
- 1 credit for each line specifying the reflective measurement model (using =~)
- *Note: The factor correlation can be skipped, due to the default to estimate this parameter.*
- **Sub-total: 3 credits**

Assessing Academic Self-Concept (16 credits)

In an international large-scale assessment, a questionnaire scale was administered to assess students' academic self-concept, that is, their perceptions of their abilities in a specific school subject. This assessment contained **six items (sc1-sc6)**. Four items were positively worded (sc1-sc4), and two items were negatively worded (sc5-sc6).

The following model describes a possible factor structure underlying this assessment (labelled **Model 2**). In this model, two correlated factors are assumed (**ScP** and **ScN**), and no mean structure is included.

Model 2



a) Explain why Model 2 represents a **reflective measurement model**.

Your explanation
All indicators are caused by the two latent variables. The latent variables (exogenous variables) are the "predictors" of the observed indicators.

SCORING:

- 1 credit for the correct explanation
- **Sub-total: 1 credit**

b) Check if **Model 2 is identified**. Ignore the mean structure.

Elements	Numbers
Number of observed pieces of information (p)	6 observed (manifest variables, sc1-sc6) with 6 variances and 5+4+3+2+1 = 15 covariances; alternative computation with 6 manifest variables: 6*(6+1)/2 ⇒ p = 21
Number of freely estimated parameters in the model (q)	To be estimated: 6 factor loadings (a-f) + 6 residual variances (v1-v6) + 1 factor correlation (r) ⇒ q = 13
Degrees of freedom of the model (df_M)	$\mathrm{df}_M = p - q = 21 - 13 = 8 > 0$
Conclusion concerning the model identification Select from: (over-)identified, exactly identified, or not identified	The model is (over-)identified.

SCORING:

- 1 credit for the correct number p
- 1 credit for the correct number q
- 1 credit for the correct degrees of freedom of the model df_M
- 1 credit for the correct decision
- **Sub-total: 4 credits**

c) Provide the **model-specifying equations of the indicators sc1, sc2, sc5, and sc6** in Model 2. Once again, ignore the mean structure.

Fill in the equations here.

Indicator variables	Model equation
sc1 =	$a \cdot ScP + h_1$
sc2 =	$b \cdot ScP + h_2$
sc5 =	$e \cdot ScN + h_5$
sc6 =	$f \cdot ScN + h_6$

SCORING:

- 1 credit per correct equation

- **Sub-total: 4 credits**

d) Provide the following elements of the **model-implied covariance matrix of Model 2**.

Fill in the results here.

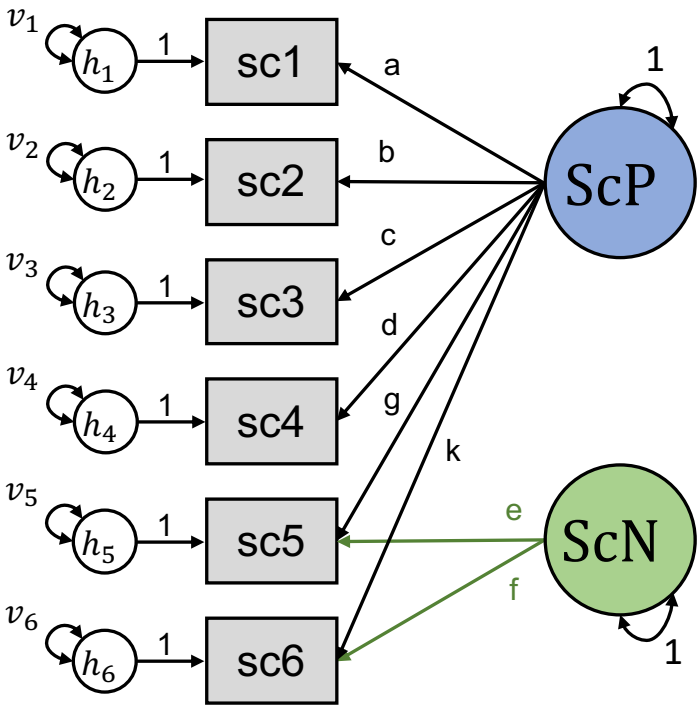
Element	Expression
$Var(sc1) =$	$a^2 + v_1$
$Cov(sc1, sc3) =$	$a \cdot c$
$Cov(sc4, sc5) =$	$d \cdot r \cdot e$
$Cov(sc5, sc6) =$	$e \cdot f$

SCORING:

- 1 credit per correct element
- Students may either use the variance-covariance rules or the path tracing rules to derive these elements.
- *Note: Students do not need to provide the derivation or an explanation how they got to the result.*
- **Sub-total: 4 credits**

The following figure shows an alternative model to describe the factor structure (labelled as **Model 3**):

Model 3



e) Provide the **model-specifying equations of the indicators sc5 and sc6** in Model 3. Once again, ignore the mean structure.

Fill in the equations here.

Indicator variables	Model equation
sc5 =	$e \cdot ScN + g \cdot ScP + h_5$
sc6 =	$f \cdot ScN + k \cdot ScP + h_6$

SCORING:

- 1 credit per correct equation
- **Sub-total: 2 credits**

f) Provide the following element of the **model-implied covariance matrix of Model 3**.

Fill in the results here.

Element	Expression
$Cov(sc5, sc6) =$	$e \cdot f + g \cdot k$

SCORING:

- 1 credit per correct element
- Students may either use the variance-covariance rules or the path tracing rules to derive these elements.
- *Note: Students do not need to provide the derivation or an explanation how they got to the result.*
- **Sub-total: 1 credit**

Intervening on Children's Mindset and Stereotypes (6 credits)

In an intervention study, Lee et al. (2021) evaluated the effectiveness of an intervention targeting children's mindsets and gender stereotypes in the context of mathematics. As part of their evaluation, the researchers estimated the following path model.

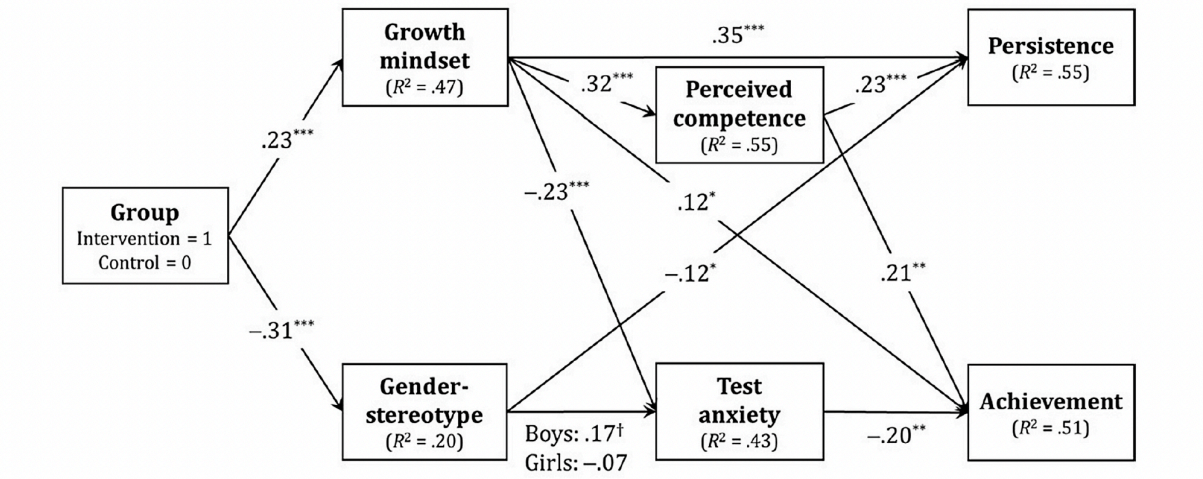


Fig. 2. Direct and indirect effects of the Intervention program. $N = 203$. For the path in which the gender difference was significant, the standardized coefficients for girls and boys are presented separately. The T1 scores of all variables, except for Group, were controlled for. Disturbance terms, control variables, and paths that were not statistically significant at $p < .10$ are not presented for clarity.
†$p < .10$. *$p < .05$. **$p < .01$. ***$p < .001$.

Moreover, Lee et al. (2021) estimated several indirect effects from this model. The following table shows the indirect effect estimates, their standard errors, and the respective *p*-values.

Table 4
Estimated coefficients for the indirect effects in the path model.

Indirect path				β	*SE*	*p*
Group →	Growth mindset	→ Perceived competence		.07***	.02	<.001
Group →	Growth mindset	→ Test anxiety		-.05*	.02	.019
Group →	Growth mindset	→ Persistence		.08**	.02	.001
Group →	Growth mindset	→ Achievement		.03	.02	.09
Group →	Growth mindset	→ Perceived competence	→ Persistence	.02***	.01	<.001
Group →	Growth mindset	→ Perceived competence	→ Achievement	.02**	.00	.001
Group →	Growth mindset	→ Test anxiety	→ Achievement	.01	.01	.055
Group →	Gender stereotypic beliefs	→ Persistence		.04*	.02	.034

*$p < .05$. **$p < .01$. ***$p < .001$.

a) Focus on the relations between **Growth mindset, Perceived competence,** and **Persistence** in this model. Which of the following statements about these relations are true or false?

Statement	True	False
All direct paths connecting these three variables are positive and statistically significant (with at least $p < .05$).	X	☐
The indirect effect of the Group variable via these three variables (Group → Growth mindset → Perceived competence → Persistence) is positive and statistically significant (with at least $p < .05$).	X	☐

The indirect effect of the path Growth mindset → Perceived competence → Persistence can be computed as 0.32*0.23*0.35.	☐	X
Concerning the path Growth mindset → Perceived competence → Persistence, there is evidence for full mediation of the relation between Growth mindset and Persistence via Perceived competence in this model.	☐	X

NOTES:

Statement 1: True, as the figure suggests.
Statement 2: True, as the table suggests.
Statement 3: False, because the indirect effect involves only two paths and is computed as 0.32*0.23.
Statement 4: False, because even if the indirect effect is statistically significant (which neither the figure nor the table suggests, although it is likely), the direct effect Growth Mindset → Perceived competence is positive and statistically significant. This suggests that, at most, partial mediation is possible.

SCORING:

- 1 credit per correct response
- **Sub-total: 4 credits**

b) The model had the following chi-square test statistic with the respective *p*-value: $\chi^2(116) = 159.8$, $p < .001$. What does this result mean for the fit of the model? Explain.

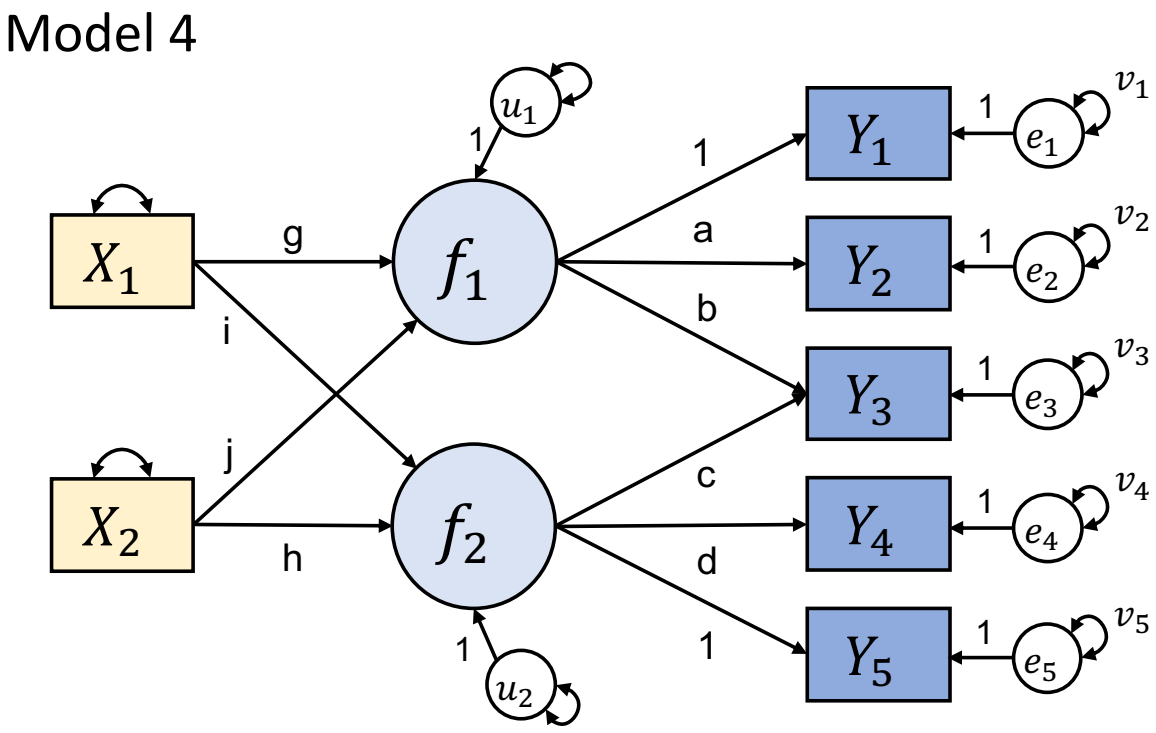
Interpretation
Significant chi-square test suggests a discrepancy between the observed and model-implied covariance matrices and thus not a good model fit.

SCORING:

- 1 credit for the correct interpretation of this finding (i.e., not a good model fit)
- 1 credit for the explanation
- **Sub-total: 2 credits**

A MIMIC Model (12 credits)

MIMIC models are so-called "Multiple-Indicators-Multiple-Causes" models and typically contain a reflective measurement model and some predictors of the latent variables (factors). The following figure shows a MIMIC model, which is named "**Model 4**".



Notice that this model does not contain a mean structure.

a) Model 4 contains **two endogenous latent variables**. Provide the equations of these two variables in the **structural part (not their measurement parts!)** of this model.

Fill in the model equations here.

Outcome variable	Model equation
$f_1 =$	$g \cdot X_1 + j \cdot X_2 + u_1$
$f_2 =$	$i \cdot X_1 + h \cdot X_2 + u_2$

SCORING:

- 1 credit for identifying f1 and f2 as the endogenous latent variables
- 1 credit per correct equation
- **Sub-total: 3 credits**

b) Model 4 contains several indirect effects. How can the **indirect effect of X_1 on Y_2** be computed?

Indirect effect
The indirect effect is the product of all parameters connecting X1 and Y2. There is only one pathway possible via f1. Hence, the indirect effect is computed as $g \cdot a$.

SCORING:

- 1 credit for the indirect effect
- *Note: An explanation is not needed.*
- **Sub-total: 1 credit**

c) The effect of $\boldsymbol{X_1}$ **on** $\boldsymbol{Y_3}$ is the **sum of two indirect effects** (i.e., two possible pathways connecting the two variables). Provide this sum.

Indirect effect
The two indirect effects are $g \cdot b$ (via f1) and $i \cdot c$ (via f2). Hence, the sum is $g \cdot b + i \cdot c$.

SCORING:

- 1 credit for each correct indirect effect.
- *Note: An explanation is not needed. Should students forget to write down the sum, but the two indirect effects are correct, only 0.5 credits are deducted.*
- **Sub-total: 2 credits**

d) Provide the **lavaan code** that specifies Model 4.

Note: You do not need to label the parameters as in the figure. Pay attention to which factor loadings are fixed to 1. Do not forget the variances of and/or covariances among the exogenous variables.

Lavaan code for the model specification

```
Model4 <-  '
           # Measurement models
           F1 =~ Y1 + Y2 + Y3
           F2 =~ Y5 + Y4 + Y3

           # Structural model
           F1 ~ X1 + X2
           F2 ~ X1 + X2

           X1 ~~ X1
           X2 ~~ X2

           '
```

SCORING:

- 1 credit for each correct measurement model (F1 and F2)

- 1 credit for each correct structural model (F1 and F2)
- 1 credit for specifying the two variances of the exogenous variables X1 and X2.
- Note: Should the order of indicators of F2 be incorrect, only 0.5 credit is deducted.
- **Sub-total: 5 credits**

e) If Model 4 also contained a **correlation between the residuals of f_1 and f_2**, named u_1 and u_2, which one line of lavaan code would you have to add to the code?

Added line of lavaan code
`F1 ~~ F2`

SCORING:

- 1 credit for the correct line
- **Sub-total: 1 credit**